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# THE PRODUCTION OF HIGH-VOLTAGE CATHODE RAYS OUTSIDE OF THE GENERATING TUBE.*

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## (1) INTRODUCTION.

SINCE the pioneer work of Lenard in bringing cathode rays out through a window in the wall of the generating tube, various modifications have been made in the tube by different workers.

Up to the present time, however, the cathode-ray tube has remained a low-voltage, low-current device when compared with the modern high-voltage X-ray tube, and has, for its operation, always had to be connected to the pump system.

It has been found possible to develop a type of cathode-ray tube which can be sealed off from the pump and is capable of operation at several milliamperes and up to at least 350,000 volts. Tubes of this type constitute so intense a source of cathode rays and so simplify cathode-ray experimentation, that they have seemed to merit detailed description.

## (2) HISTORICAL.

The original Lenard tube [1] is shown in Fig. 1, in which *K* is the aluminum cathode, *AA* the anode, *mm* a perforated brass cap cemented to the end of the discharge tube opposite the cathode, and *V* a metal diaphragm. The window is of aluminum foil 0.0026 mm. thick cemented with marine glue over the 1.7-mm. opening in the metal cap *mm*. The tube was operated

* Presented at the Stated Meeting held Wednesday, October 20, 1926.

[1] P. LENARD, *Ann. d. Phys.*, 51, pp. 225–267 (1894).

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from an induction coil and the most favorable voltage was that corresponding to a 3-cm. spark between small metal spheres. The air in front of the window showed luminosity for a distance of 5 cm., and fluorescent materials showed luminosity at distances as great as 6 or 8 cm. from the window.

Des Coudres,[2] in 1897, described a cathode-ray apparatus in which the tube itself differed from that of Lenard mainly in this, that the body of the tube was made of hard rubber. For excitation, he used a high-frequency coil instead of the ordinary induction coil.

FIG. 1.

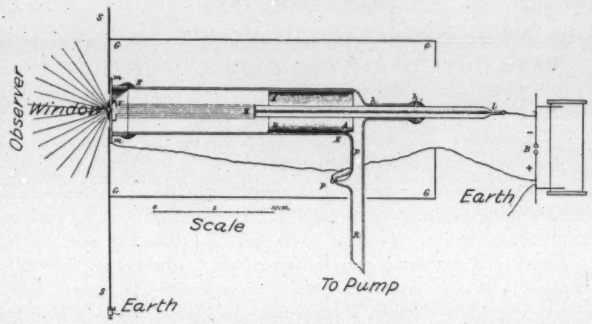



Diagram of original Lenard tube.

Pauli,[3] in 1910, described a new window construction consisting of a sheet of aluminum foil cemented to a brass plate having twenty 2-mm. holes, 1 mm. apart, which, for better support of the aluminum foil, were covered with copper gauze.

In 1920, Pauli [4] described a radical departure from the early tube of Lenard, consisting in the substitution, for the thin metal foil, of a tiny hole, 0.01 mm. in diameter, in a sheet of platinum. A similar perforated platinum plate was placed behind this, and two pump systems were employed, one connected to the main portion of the tube, containing the cathode, and the other to the smaller chamber between the two diaphragms.

In 1925, Krüger and Utesch [5] described a tube with a high

[2] TH. DES COUDRES, *Ann. d. Phys.*, **62**, pp. 134–144 (1897).
[3] PAULI, *Ztschr. f. Instrumentenkunde*, **30**, pp. 133–137 (1910).
[4] PAULI, *Phys. Ztschr.*, **21**, p. 11 (1920).
[5] KRÜGER and UTESCH, *Ann. d. Phys.*, **78**, pp. 113–156 (1925).

vacuum and hot cathode. (They said that such a tube had also been used by O. Eisenhut and referred to an article by him, "Uber glühelektrisch ausgelöste Kathodenstrahlen, Untersuchung derselben und ihre Verwendung zu messenden Versuchen bei grossen Strahlengeschwindigkeiten," Heidelberg, Dissertation, Mai, 1921. The writer has been unable to get this.) Their window consisted of aluminum foil, 0.011 to 0.005 mm. thick, cemented with picein to a perforated water-cooled brass plate. The perforations consisted of sixty-five holes 0.8 mm. in diameter. The tube was operated from a 500 ~ high-tension transformer with a single kenotron rectifier, using peak voltages up to 60,000.

In 1926, Peters and Schlumbohm described in *Naturwissenschaften* of July 23 (pp. 218–219) a tube embodying the principle of the 1920 tube of Pauli. Their tube had a hot cathode, a diaphragm and seven orifices 1 mm. in diameter, all accurately lined up. Twelve vacuum pumps were connected to the chambers between the various orifices of this tube.

### (3) GENERAL DESCRIPTION OF THE TUBE.

Experimental models of this type of tube have been made of various sizes ranging in overall length from 53 to 155 cm.

Fig. 2 shows a model which has been used for voltages up to 250,000 (max.).

The cathode consists of a flat spiral of tungsten wire placed in a hemispherical focusing cup, *c*.

The anode proper, *a*, is at the same time the window, and consists of a sheet of nickel foil 0.0127 mm. thick and 7.5 cm. in diameter, supported against the pressure of the atmosphere by a honeycomb structure of molybdenum, *b*. The edge of the window is soldered to the invar ring, *r*, and this in turn is sealed directly to the end of the glass anode arm, at *s*.

A long copper tube, *k*, electrically connected to the window, serves as an electrostatic shield to protect the glass of the anode arm from puncturing.

This shield is very essential to high-voltage operation as, without it, full cathode potential may build up on the inside of the glass anode arm, even in the immediate neighborhood of the anode, resulting in a potential gradient which will cause destructive disruptive discharges through the glass.

As a result of the presence of the shield, very few electrons can get to the glass walls of the tube. For most of the electrons constituting the cathode-ray stream go right through the window, and, of those reflected from the window and the window support, nearly all will go to the shield and only a relatively very small

FIG. 2.

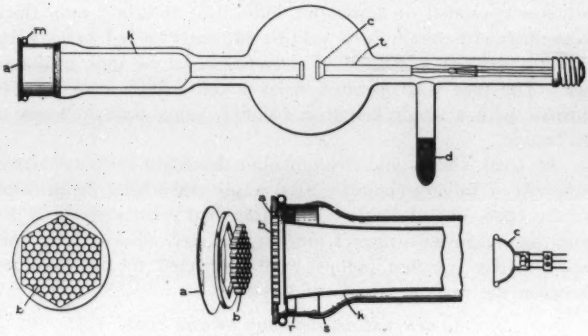



Diagram of new cathode-ray tube.

number can emerge from the end of the shield facing the cathode against the hostile field of the latter.

The bulb is 20 cm. in diameter and the overall length of the tube is 89 cm.

A model similar to this, but with a 30.5-cm. bulb and an overall length of 155 cm., has recently been used for voltages up to 350,000 (max.).

#### (4) DETAILED DESCRIPTION OF THE TUBE.

(1) *The Cathode.*—The design of the cathode is important in that it greatly affects the size of the cathode beam, at the point where the latter impinges upon the window (the focal spot), and plays a dominating rôle in determining the evenness of distribution of energy over the focal area. These two factors, size of focal spot and evenness of distribution of energy, are in turn important as they determine the maximum temperature of the window and so set a limit to the amount of current which can be passed through the tube.

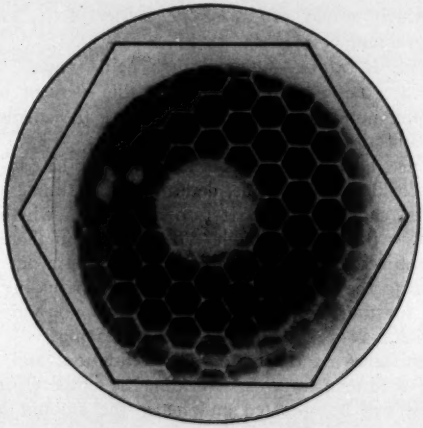
It is desirable that all of the cathode-ray beam shall fall upon the window, as any portion of it striking the copper shield is not

only lost so far as cathode-ray output is concerned, but causes needless heating of the various parts in the anode arm and needless X-ray production. The determination of cathode-ray output from the tube is also facilitated by having the entire cathode-ray beam fall upon the window.

The tungsten filament is 0.0216 cm. in diameter and is coiled into a flat spiral 0.5 cm. in diameter, mounted as shown in Fig. 2, in a recess in the centre of a hemispherical molybdenum cup, *c*, 2.5 cm. in diameter.

If the end of the copper tube facing the cathode were closed,

FIG. 3.



Focal spot picture of new cathode-ray tube.

the focal spot, with the present cathode design, would be about 3 mm. in diameter, but the electrostatic field extends into the open end of the tube and this results in such a spreading of the cathode-ray beam that the actual focal area on the window is about 6 cm. in diameter.

To determine the size and location of the focal spot and the distribution of energy over it, a piece of ordinary blue-print paper is placed in contact with the window and the tube is operated for perhaps a quarter of a second. (Silver bromide emulsions are far too sensitive to be conveniently used for this purpose.)

Fig. 3 shows such a focal spot picture. In this case it will

be noticed that the periphery of the focal spot is much sharper at the left than it is at the right. This is due to a faulty location of the copper shield which has intercepted a portion of the cathode-ray beam on the left. The figure shows convolutions corresponding to the cathode filament spiral. It also shows a hollow centre; but this is preferable to the hot spots which may easily be produced by changing the cathode design so as to avoid the hollow centre.

The front edge of the focusing device is given a 1.5-mm. radius and the cathode support wires are covered with a molybdenum tube, *t* (Fig. 2), to prevent cold-cathode discharge from regions which would otherwise be subjected to too high a potential gradient.

(2) *The Window.*—Other things being equal, one would choose as a window material that metallic element having the lowest atomic number, as this would involve the smallest loss by reflection and absorption.

There are, however, several other factors to be considered.

The ductility, for example, determines the thinness to which the material can be rolled. It also determines whether or not the window will stretch enough, when placed on the supporting grid and subjected to the atmospheric pressure on one side only, to conform to the shape of the grid without wrinkling. This is important for the reason that wrinkles mean non-uniformity of stress, which may well lead to subsequent tearing.

The resistance of the material to oxidation, and its elastic limit at elevated temperatures, determine the temperature at which the window can be operated and hence the amount of current which can be passed through the tube.

The heat conductivity determines the lateral heat flow to the window support and hence, to a certain extent, determines the amount of current which can be passed through the tube.

The question of whether the material can be easily soldered or brazed is important. The alternative method of cementing the window to the support with some high-melting wax, as de Khotinsky cement, is not good, as this deteriorates rapidly under vigorous cathode-ray bombardment. Furthermore, a wax joint is not desirable in a tube which is to be sealed off from the pump.

For high-voltage operation, even on the pump, it is necessary

that the window should be free from holes. Even a single hole that can, with the most favorable illumination, be seen with the microscope, is fatal to operation with as much as 200,000 volts.

Aluminum foil was first used. It was attached, at the periphery, to the invar support ring by means of de Khotinsky cement. There were several serious objections to these first windows. The low softening temperature of the cement limited the allowable enery input of the tube; the bombardment by electrons, which had been reversed in direction by collision with the air, decomposed the cement, filling it with gas bubbles until it no longer held the vacuum; and finally the elastic limit of aluminum falls to very low values at temperatures easily reached by the window.

Commercial copper was next tried, and it was found that with care it could be rolled down to foil 0.0127 mm. thick and free from holes. Windows of this material were attached at the periphery to the support ring by soft solder, which gave a joint which did not deteriorate under electron bombardment. In some cases these copper windows developed numerous holes in use. Experiment has since shown that such material, if heated strongly in a reducing atmosphere, develops innumerable holes. This is due to the original oxide content of the copper.[6]

It was finally found possible to roll nickel down to good foil, 0.0127 mm. thick, and this has been by far the most satisfactory window material.

The procedure followed in making this nickel foil is as follows:[7]

[6] It has more recently been shown that 0.0127-mm. foil made from special copper, freed from oxide by the addition of silicon, does not develop holes on heating. Such foil should be serviceable as a window material.

[7] As one alternative method of making foil suitable for windows, it seemed possible that the method employed by the gold beater might be better than rolling. Through the courtesy of Mr. A. M. Fraser, of New York, a manufacturer of beaten gold and other metal leaf, we were able to make the following test: Good rolled sheets of annealed copper, nickel and platinum, 0.0254 mm. thick and free from holes, were submitted to Mr. Fraser to be beaten to about half their thickness. All of this beaten material was found to be hopelessly full of holes.

Another alternative method consists in electroplating. Splendid looking nickel foil has been produced in this way, by plating from a hot solution of nickel chloride on to a highly polished sheet-molybdenum cathode, taking special care to guard against loose particles in the bath. Foil even as thin as 0.0025 mm. strips easily from the molybdenum cathode, and shows very few holes. Electro-deposited material has not yet been tried out in windows.

Commercial sheet nickel, 0.127 mm. thick, made by the American Nickel Company, of Clearfield, Penna., is cut into rectangular sheets, 6.5 × 10 cm., and annealed in hydrogen. Eight of these sheets are then placed inside of a folded sheet of monel metal, 0.063 cm. thick and highly polished, and the pack is then cold rolled, in a mill having carefully ground rolls, down to where the sheets of foil are roughly square. The sheets are then removed from the pack, again annealed in hydrogen, and then cut in two and put into a fresh envelope of monel metal like the first one and then rolled down to the desired thickness of 0.0127 mm.

Before inspection, the sheets are first heated in a vacuum to a temperature of about 750° C. This vacuum firing is important as it seems to open up holes which were originally present but temporarily closed, perhaps by foreign matter which distils out during the heating operation. Experience has shown that if nickel foil which has not had the preliminary firing operation is used for windows, holes may develop as the window is heated by electron bombardment during the exhaust.

The most satisfactory method found for inspecting a sheet for holes consists in taking it to a photographic dark-room and placing it over an opening in an illuminating box containing a powerful light source, and then, with eyes in good condition, going over it either with a low-power magnifying glass or with the naked eye.

It at first seemed probable that foil showing an occasional hole which could be easily recognized by the above inspection method would show others recognizable, by this method, only with great difficulty, and still others which could not be so recognized but which would make their presence known, when the sheet was used as a window, through lack of vacuum tightness in the tube. Experience, however, shows that in general a hole which is recognizable in any way is easily distinguished by the above method.

Many sheets of foil will be found free from holes. Where a small hole is found, it is closed with soft solder.

(3) *The Window Support.*—The window support consists of a honeycomb structure of molybdenum, *b*, in Fig. 2. It is made of sheet molybdenum, 0.015 cm. thick, bent up into hexagonal cells which are all fastened together and to the surrounding molybdenum frame by copper brazing in hydrogen.

The individual cells measure 0.6 cm. between opposite sides and are 0.47 cm. deep.

Molybdenum is especially well adapted to this purpose because of the fact that it expands and contracts so little as its temperature changes from that of the room to that assumed by the window support when the tube is in operation. This is important if the tube is to be operated with as high current values as possible. While both the window and the support heat up under bombardment, they do so at very unequal rates and, similarly, they cool at very unequal rates. This leads to a certain amount of lateral movement of the window with respect to the support and, with currents sufficient to heat the window to visible redness, to a temporary wrinkling of the window. Experience teaches that it is permissible to have at times a surplus of window material with respect to the support but that at no time during the heating or cooling process should there be a dearth of window material, as this puts the thin foil under excessive tension and may rupture it. This last condition has been experienced with invar supports. (Invar at room temperature has a lower temperature coefficient than that of molybdenum, but upon being carried from room temperature to, say, 400° C., its total expansion is much greater than that of molybdenum.)

The shape of the openings in the window support seems to be important if the maximum allowable current is to be as high as possible. Experiment has shown that with a molybdenum support having circular openings with a diameter of 0.6 cm. (equal to the distance between opposite sides of the hexagons in Fig. 2), the window material would tear with tube currents which could be safely borne when using the molybdenum support with hexagonal openings, and the space factor was more favorable in the latter than in the former case.

The individual cells of the molybdenum support member are made as large as they are, and from such thin material, in the interest of efficiency. By calculation, about 80 per cent. of the window is free, in the sense of being directly exposed to the electron bombardment.

(4) *Attachment of the Window to the Window Frame.*—The window is attached to the invar frame with soft solder, the two opposing surfaces having been first carefully tinned. The most scrupulous care is necessary to avoid the presence of dust

particles between the window and its honeycomb support, as very moderate-sized dust particles, if present, will often puncture the window as soon as the vacuum is applied to the tube.

(5) *The Shield.*—The shield, *k*, in Fig. 2, is made of thin copper and, at the end facing the cathode, the edge is given a radius of 1.5 mm. to prevent cold-cathode discharge during the half cycle when it is at a high negative potential with respect to the cathode structure. It is so designed as to intercept none of the primary cathode-ray beam.

(6) *The Exhaust.*—The tube is exhausted by means of a Langmuir condensation pump, with suitable auxiliary pumps and a liquid-air trap between condensation pump and cathode-ray tube.

All of the metal parts, including the window, are vacuum fired before being put into the tube.

After a suitable vacuum has been reached, the glass is strongly heated to free it from gas. The tube is then allowed to cool slightly and then a bombardment of the shield is started. This is made possible by the use of a solenoid or bar magnet to deflect the cathode rays away from the window on to the shield. For this operation the window support, with the soft solder joint used for the window attachment, is cooled by tap water flowing through the single turn copper coil, *m*, of Fig. 2, and the tube is operated from a high-voltage system, which is so arranged that the positive terminal can be grounded. The bombardment is continued until the shield is visibly red in the dark.

The window is then bombarded for a few minutes, care being taken not to heat it to the oxidation temperature.

The tube is then ready to use on the pump.

If it is intended to be sealed off from the pump, the tube is provided with a side tube, *d*, in Fig. 2, containing either cocoanut charcoal or a small rod of calcium in a tungsten heating spiral.

If the charcoal is used, it is given a preliminary vacuum firing and, while the tube is connected to the pump, is heated as strongly as the glass will stand, until most of the residual gas has been removed. After sealing the tube off, it may be found to operate satisfactorily without cooling the charcoal side tube. If not, the latter is immersed in liquid air whenever the tube is used.

Calcium appears to be just as satisfactory as charcoal in the side tube and has the advantage of rendering it unnecessary to use liquid air after the tube has been sealed off from the pump.

It also does away with the time lag involved in the use of the liquid air in case the charcoal tube has been allowed to stand for some time at room temperature. (This time lag may amount to as much as a half hour.) It also has the great advantage of allowing a given tube to be used in any desired position.

### (5) SUITABILITY OF VARIOUS HIGH-VOLTAGE SOURCES FOR OPERATING THE TUBE.

The behavior of high-voltage X-ray tubes has been shown[8] to be so dependent upon the electrical characteristics of the high-voltage source that it has seemed worth while to see how these characteristics affect the performance of the cathode-ray tube.

(1a) *Induction Coil and Mercury Turbine Interrupter.*—The first work was done with an induction coil, driven by a mercury turbine interrupter, and containing originally a point-plate valve tube between the two halves of the high-tension circuit. Experiment showed that the cathode-ray tube behaved just as well without the valve tube and the latter was therefore omitted.

By means of two sphere-gaps and a kenotron, connected as shown in Fig. 4, the "useful" and "inverse" voltages were measured, with the results shown in the following table:

| | Kilovolts. | |
|---|---|---|
| M.A. | "Useful." | "Inverse." |
| 1.6* | 200 | 117 |
| 2.0 | 250 | 127 |
| 2.0 | 275 | 165 |
| 2.0 | 300 | 191 |

* This was the maximum current that could be obtained at this voltage, owing to space charge resulting from the design of this particular cathode.

As the tube has operated well under these conditions, it is clearly able to handle a very considerable "inverse" voltage.

(1b) *Induction Coil and Electrolytic Interrupter.*—Up to at least 200,000 volts, the tube operated very nicely when the Wehnelt electrolytic interrupter was substituted for the mercury turbine.

(2) *Transformer.*—The experiment was tried of operating the tube directly from a 60 ~ high-voltage transformer. With such half-wave operation of a transformer, the "inverse" will be greater than the "useful" voltage by an amount determined

[8] COOLIDGE and KEARSLEY, *Am. Jour. Roentgenology*, 9, pp. 77–101 (1922).

by the load, the design of the transformer and the electrical characteristics of the supply circuit. If the high-tension voltage is controlled by a rheostat in the primary circuit the "inverse," corresponding to a given milliamperage and a given "useful" voltage, will increase with the amount of resistance used in the primary. Under these conditions, it is then the "inverse" voltage that sets a limit to the "useful" voltage which can be obtained with a given transformer. For the above reasons it is, in general, better to keep the resistance of the external circuit as

FIG. 4.

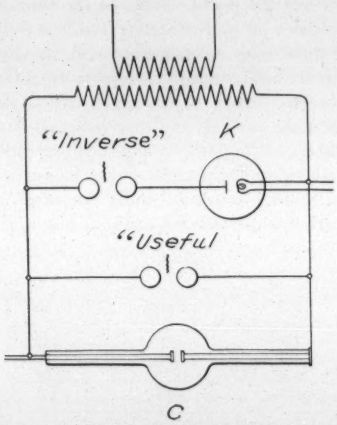



Diagram of connections for measuring "useful" and "inverse" voltages.

low as possible and use auto-transformer control of voltage, so as to keep the "inverse" practically the same as the "useful." Upon doing this, it was found that the cathode-ray tube would rectify its own current up to the safe limit of the transformer in question, 250,000 volts. This method of operation is, however, not to be recommended for currents of only two or three milliamperes, as it makes the high-tension circuit capable of delivering, in case of trouble, such large amounts of energy. This is characteristic of half-wave rectification and applies equally well if a kenotron is used in series with the cathode-ray tube.

(3) *Transformer with Direct Current Superimposed on*

*Alternating-current Excitation.*—By the use of direct current in the primary, it has been found possible to reduce the "inverse" for a given "useful" voltage.

The diagram of connections is shown in Fig. 5 in which $R_1$ and $R_1^1$ are rheostats used in the low-tension circuit to control the high-tension voltage, and $R_2$ and $R_2^1$ rheostats to control the direct-current excitation. $L$ and $L^1$ are open-magnetic-circuit inductances used to reduce the flow of alternating current through the direct-current source.

The tube behaves nicely on the above system and, with the conditions employed, it was found that, with a tube current of

FIG. 5.

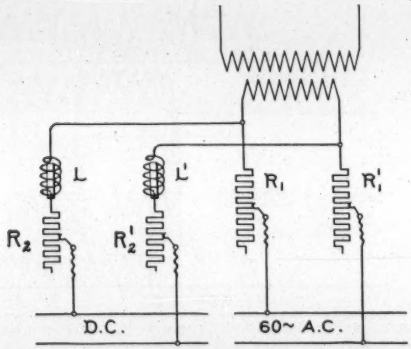



Diagram of connections for transformer with direct current superimposed on alternating-current excitation.

2 ma., and a "useful" voltage of 250,000 the "inverse" was 180,000. In other words, the "inverse" was higher in this experiment, with respect to the "useful," than it had been in the case of the induction coil, but was still well below the "useful."

A modification of the above system, shown diagrammatically in Fig. 6, has also been tried. In this modification the direct-current source has been replaced by a direct-current load consisting of a Tungar rectifier bulb, $T$, and a rheostat, $R$. The Tungar circuit must be, by trial, so connected that it utilizes the half-cycle which is not taken by the tube.

Both of the above systems are good and where both A.C. and D.C. sources are available, there is little to choose between

them. The latter system will, however, recommend itself where only A.C. is available.

(4) *Transformer with Mechanical Rectifier.*—A 200,000-volt transformer with a rectifying switch driven by a synchronous motor was available. This outfit gives full-wave rectification and therefore permits of the use of any desired amount of ballast resistance. The tube ran very nicely on this machine but, although the tube was in this way relieved of the necessity of doing any rectification work whatsoever, it was not apparent to the eye that it behaved any better than when operating from other

FIG. 6.

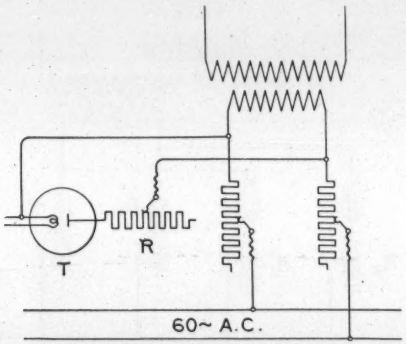



Diagram of connections for transformer with Tungar rectifier load superimposed on primary.

high-voltage sources where it had to deal with 100,000 or even 200,000 volts "inverse."

(5) *Constant-potential Continuous-current.*—The use of a direct-current source is attractive for many reasons. Some of the author's work up to 100,000 volts has been done with a 2000 ~ transformer-kenotron-condenser outfit.

(6) *Generating System with Positive Terminal Grounded.*—It is a great convenience in cathode-ray work to have the window of the tube grounded, as this makes it possible to have the tube and all other parts of the high-tension circuit inside of a grounded metal inclosure, while giving ready access to the window. It also permits of cooling of the window frame with flowing tap water.

Experience shows that, by the use of suitable insulation transformers, the various high-voltage sources which have been referred to can be easily made to permit of grounding one terminal.

In the case of the induction coil, the insulation transformer

FIG. 7.

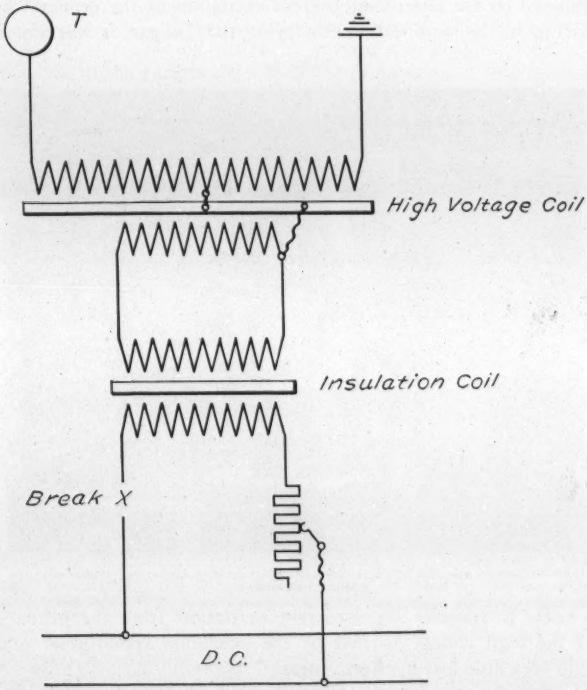


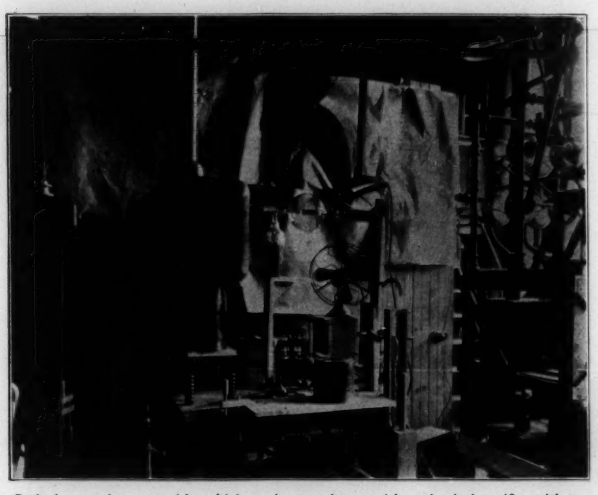

Diagram of connections for induction coil with insulation coil in primary and one high-tension terminal grounded.

becomes a second induction coil, in which the core and primary and high-voltage insulation may be made just the same as in the first coil, with a secondary having only as many turns as the primary. It is essential that a lead should be brought out from

the middle of the secondary of the high-voltage coil and connected to the core and primary of this coil, as shown in Fig. 7. The core and primary of the high-voltage coil must be well insulated from ground. To prevent corona losses, it may also be necessary to use a larger terminal, *T*, on the high-voltage coil.

In the case of the transformer with direct current superimposed on the alternating-current excitation of the primary, as well as in the modification employing the Tungar, it was found

FIG. 8.



Cathode-ray tube operated from high-tension transformer with mechanical rectifier, with one terminal grounded.

possible to transfer direct-current excitation from the primary of the high voltage to that of the insulation transformer and with very little loss in effectiveness.

With the transformer and mechanical rectifier, the whole outfit was mounted on an insulated platform and the insulation transformer placed on the floor below the platform. One additional change was necessary, in that the synchronous motor had to be operated from a 1 : 1 insulation transformer (the alternative would have been to insert between the motor and the rotating

switch, a long insulating shaft) and the motor circuit connected to the case of the high-tension transformer. This installation is shown in Fig. 8.

The 200,000-volt constant-potential continuous-current outfit used has one grounded terminal.

### (6) OPERATION OF TUBE.

(1) *Control of Cathode-ray Intensity.*—Cathode-ray intensity can of course be varied by varying the distance from the window to the experimental material in question. This method, however, has the disadvantage of changing electron velocities at the same time.

In the first experiments tried with the tube, it was not easy to satisfactorily change the intensity over wide limits by changing the temperature of the hot cathode, and for the following reasons: The early tubes carried considerable current at high voltages even when the cathode was cold. This cold-cathode discharge was a highly variable quantity and would sometimes amount to as much as two or three-tenths of a milliampere. Furthermore, the fraction of these electrons reaching the window was a rapidly fluctuating quantity. The situation was improved by insulating the shield from the window so that a milliammeter connected to the window would respond only to that portion of the cathode-ray discharge which struck the window. It was still unsatisfactory, however, for work of any considerable degree of precision with tube currents of appreciably less than a milliampere.

An attempt was made to reduce intensity by placing close to the window a thin sheet of brass which, with the help of a sewing machine, had been perforated with tiny regularly spaced holes. From the measured diameter of the holes and the spacing, it was easy to calculate the free-opening factor of such a screen. Furthermore, at a distance of as much as 1¼ cm. in front of the screen the individual tiny cathode-ray beams which had come through the openings in the screen were no longer distinguishable (scattering, of course, tends to make the intensity uniform over the cross-section of the beam). According to a photographic test, however, the actual screen factor was of a different order of magnitude from that calculated from the size and spacing of the holes. The explanation of the discrepancy doubtless lies in the multiple reflection of the electrons, which must take place between the screen and the window. This multiple reflection is unfortu-

nate in that it not only changes the screen factor, but also changes the distribution velocities, so that one could not assume that, with the screen, he had the same velocity distribution as without it, even though the distance of the point in question from the window were the same in both cases.

A rotating disc with a small opening was also tried. It was used in conjunction with a photographic shutter which served to prevent the window from being uncovered more than once. This method, however, labors under the same difficulty as the preceding one; but the effect of multiple reflection can, in this case, be reduced by the use of a large opening and correspondingly high speed.

The design of tube described in this article shows so little cold-cathode effect that it permits of satisfactorily changing intensity over a very considerable range through regulation of filament temperature.

(2) *Protection of Operator.*—The question of protection for the operator, from the radiations coming from the tube, is very important. One has to think not only of the cathode rays themselves, but also of the X-rays coming from the copper shield, the window support, the window and any matter outside of the tube which is within the range of the cathode rays. In connection with the X-rays, one must remember that he is here dealing with a very wide spectrum, including, at the one end, the shortest wave-length producible by the voltage in question and, at the other end, rays of so long wave-length that they could not get through the glass walls of the ordinary X-ray tube.

(3) *Protection of Window.*—In experiments in which the window is exposed to the air, the allowable current input can be increased by directing a stream of air from a fan or blower on to the window.

The window cannot safely be used at a temperature at which appreciable oxidation takes place, as such oxidation if it extends through the window makes it porous. The window must be protected from mechanical injury. It must also be protected from a spark discharge from any nearby conducting body, as a visible spark discharge to the window appears to result invariably in a perforation. This last contingency is easily avoided by having any conducting body, which is near the window, electrically connected to it. Both the mechanical and electrical hazards can be

eliminated at the same time by permanently mounting a wire gauze frame in front of, and electrically connected to, the window.

If the tube is not to be sealed off from the pump but is to be operated only while connected to the latter, it is desirable that at least a rough vacuum should always be maintained. Otherwise dust particles may get in between the window and the window support and, if large enough, will make holes in the window when the vacuum is again put on.

The chemical action of the rays cannot safely be tried on liquids applied directly to the window, for, as a result of chemical change, it may well happen that no suitable solvent can be found for removing the resulting product.

(4) *Operating Temperature of Window.*—By the following method, it has been found possible to study, at least roughly, the temperature attained by the hottest portions of the window. A very dilute solution of a pure crystalline substance having a sufficiently high melting-point is painted on the window, with a camel's hair brush, and the solvent is allowed to evaporate. The tube is then operated with a definite current for a definite time, and the window is inspected. If at any point the tiny crystals have melted, and spread as a liquid film over the surface, the changed appearance of the window will show it. If the melting-point has not been reached, the experiment is repeated with a higher current or a longer time. In this way the operating conditions are determined which will just bring the hottest portions of the window to the melting temperature of the substance in question.

To guard against a possible change in melting-point as a result of raying, the melting-point of some of the rayed material is determined by the capillary tube method.

Malonic acid has been found satisfactory and has been applied to the window in the form of a saturated grain alcohol solution. Its melting-point, as measured after raying, was 132° C. At 100 KV., the above temperature was reached, by the hottest portions of the window, with 0.7 ma., in five seconds; at 200 KV., the same temperature was reached with 0.6 ma., in five seconds.

Potassium chlorate was also satisfactory and was applied by first wetting the window with grain alcohol and then blowing the finely divided chlorate from an insect-powder duster on to the window. In this case, the melting-point of the material after

raying was found to be 340° C., and this temperature was reached with 200 KV. and 1.3 ma., in ten seconds.

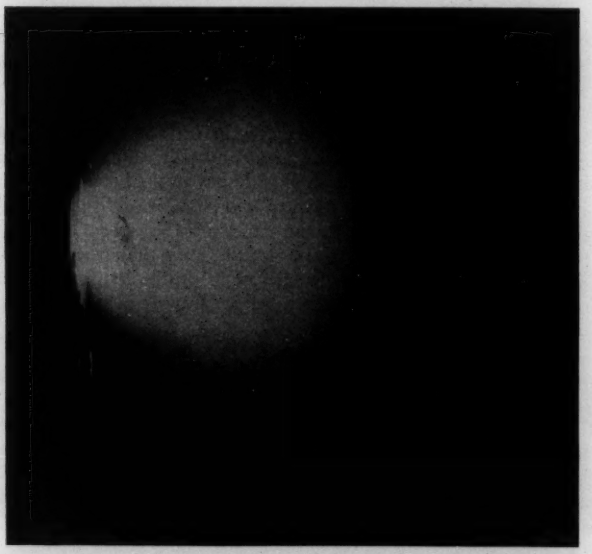
In the preceding temperature determinations, no forced air cooling of the window was employed.

(5) *Noise Emanating from Tube.*—On D.C. excitation, the operation of the tube is quite noiseless; but on 60 ~ A.C. there is a dull roar, due, presumably, to the thermophonic action of the discharge upon the air in front of the window.

### (7) DISTRIBUTION OF CATHODE RAYS IN FRONT OF WINDOW.

(1) *As Indicated by Luminosity of Air.*—Inside the tube the cathode rays approach the window in a conical beam. Some

FIG. 9.



Luminosity of air in front of window of cathode-ray tube.

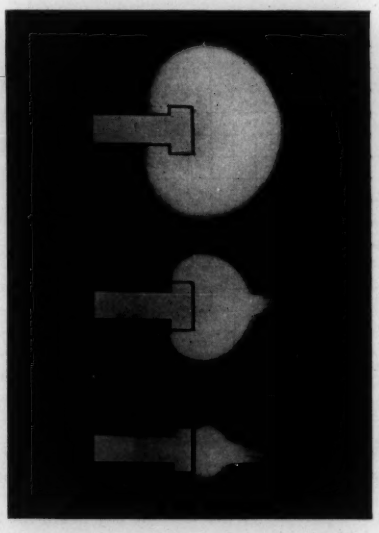
scattering of this beam takes place in the window and then a great deal more in the air outside.

This scattering is shown in Fig. 9, which is reproduced from

an ordinary photograph of the luminosity of the air in front of the window. In this case the window frame was a metal ring about 18 cm. in diameter—so large that very few electrons were scattered back of the plane of the window.

(2) *As Shown in Direct Action of the Rays on Photographic Film.*—In this case a different tube from that referred to in the

FIG. 10.



Record of cathode rays obtained by direct action of the rays on photographic film.

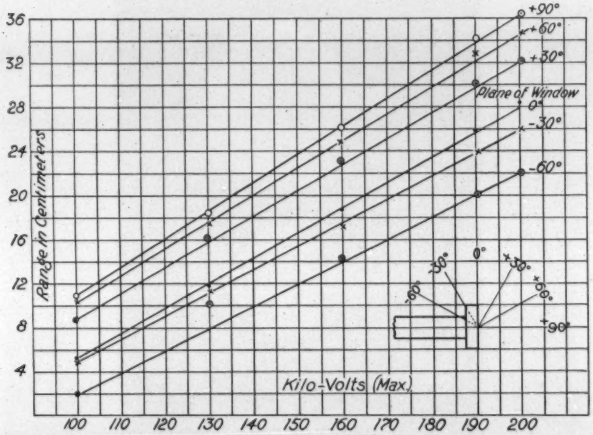
preceding paragraph was used. This tube had a smaller window, only 3 cm. square, and a much smaller window frame. The anode arm of this tube is shown to scale in Fig. 12. The window was of aluminum, 0.0254 mm. thick.

When, in a carefully darkened room, a bare photographic film is placed with one edge against the window and with its plane perpendicular to the window and passing through the centre of it, and the tube is then operated, a photographic record is obtained which shows the approximate range of the cathode rays in various

directions in front of the window. Furthermore, by suitably tailoring the film so that it can be fitted over the anode end of the tube, the photographic record can be extended so as to include also what goes on behind the plane of the window.

Fig. 10 shows three such records, the lower for 70,000 volts, the next for 100,000 and the upper for 130,000 volts. (The horizontal protuberances extending to the right in the lower two records should be disregarded, as they were caused by X-rays

FIG. 11.




Graphs showing range of cathode rays, as given by fluorescence of lime. Window of aluminum 0.0254 mm. thick.

produced when a portion of the cathode-ray beam hit the inside of the copper shield in this early tube.)

(3) *As Shown by Range Measurements with Lime.*—The boundaries of the space traversed by the electrons outside of the tube may also be easily mapped out by means of the luminescence of a piece of lime (CaO).

In a well-darkened room (the tube must be well covered to prevent the escape of light from the cathode spiral) a position is easily found at a certain distance in front of the window at which a piece of lime just begins to show the characteristic orange luminescence. At a greater distance, it appears feebly luminous, due to X-rays.

This has been done, for different voltages and in different directions, for the tube referred to in the section just preceding this, with the results indicated by the graphs of Fig. 11, in which the diagram shows the significance of the angles. The tube was operated from an induction coil, and the voltage (peak value)

FIG. 12.

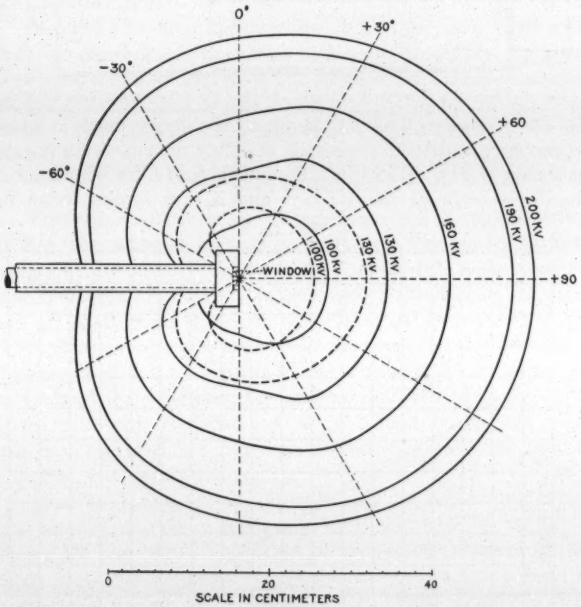



Diagram showing range of cathode rays, as obtained by means of lime and by direct action on photographic film. Window of aluminum 0.0254 mm. thick.

was measured with a sphere gap. The filament current was kept constant throughout the series, and the milliamperes varied from about 0.4 at 100 KV. to about 0.6 at 200 KV. Range has been measured straight to the centre of the focal spot, regardless of the actual path that the electrons have taken.

The results of Fig. 11 are shown in a different way by the full lines of Fig. 12. In this figure, the dotted lines for 100 and

130 KV. are taken from the photographic film records of Fig. 10. The check is not perfect, but one sees that the two methods have given results having the same order of magnitude.

A refinement which is indispensable, if the method involving

FIG. 13.

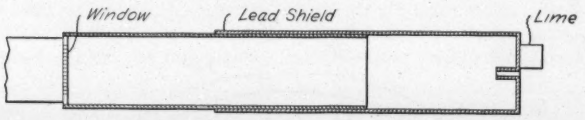
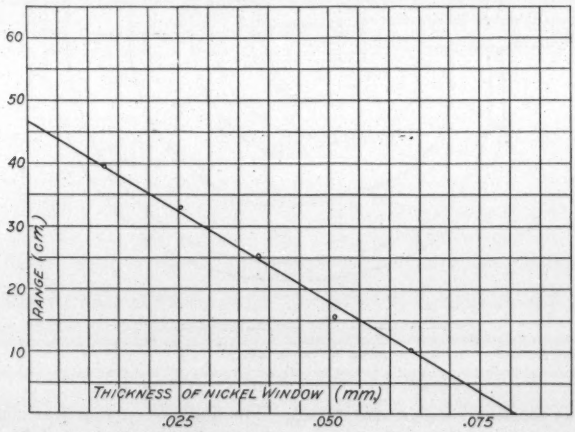



Diagram of apparatus for measuring range of cathode rays, with complete X-ray shielding.

the fluorescence of lime is to be used for voltages much in excess of 200,000, consists in completely shielding the lime from X-rays, as shown in Fig. 13, in which the lime is seen to be placed out of the direct beam of cathode rays and X-rays coming from the

FIG. 14.




Graph showing effect of window thickness on range in air of 200,000-volt cathode rays.

window. After passing through the telescoping lead shield and out through the rectangular diaphragm the cathode rays are scattered, and so get to the lime, while the X-rays do not.

(4) *Effect of Window Thickness on Range.*—The effect of the thickness of the nickel window on the range of the cathode

rays in the air has been studied for 200,000-volt operation of the tube. The range was observed by means of the fluorescence of lime, and the effective window thickness was increased at will by simply placing additional pieces of nickel foil in front of, and close to, the window. The results are plotted in Fig. 14.

By extrapolation, one sees that the range in nickel at 200,000 volts is 0.081 mm.

One also sees, from the intercept on the other coördinate axis, that, for this voltage, the range in air, if the window thickness were zero, would be 46.6 cm. The range in air with zero window thickness would then be, at 200,000 volts, about 18 per cent. greater than it is with the 0.0127-mm. window.

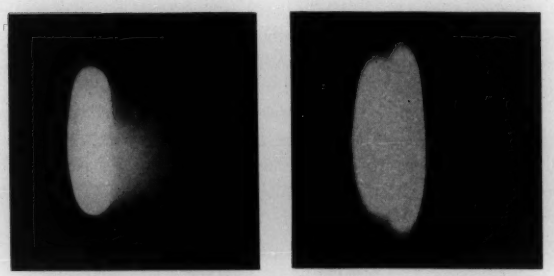
### (8) EMISSION OF X-RAYS BY AIR AND OTHER SUBSTANCES IN FRONT OF WINDOW.

That the air in front of the window emits X-rays is shown by Fig. 15*a*, which is reproduced from an X-ray pin-hole camera picture of the air and the window. The pin-hole camera in this case consists merely of a lead box with a pin-hole in the front and an X-ray film, in a light-tight envelope, facing this in the back. The pin-hole was 0.5 mm. in diameter and was placed at a distance of 46 cm. to one side of the tube axis and about 7.5 cm. in front of the window. The camera was pointed at the centre of the focal spot. The film was 23 cm. behind the pin-hole. The tube was operated at 1 ma. and 200,000 volts and the time of exposure was twenty minutes.

That the X-rays coming from the air are not due to X-rays emanating from the window and then scattered by the air is shown by Fig. 15*b*. This is an X-ray pin-hole camera picture made in exactly the same way as the first except that a sheet of aluminum 0.04 mm. thick had there been placed directly in front of the window. This thickness of aluminum was sufficient to stop the cathode rays, but not sufficient to appreciably reduce the intensity of X-rays coming from the window. Its use is seen to have prevented any X-ray production from the air.

Fig. 16 is reproduced from an X-ray pin-hole camera picture of a collection of fifteen different metals arranged in one plane as shown in Fig. 17 and placed with this plane parallel to the window and 5 cm. in front of it. In this case the tube was

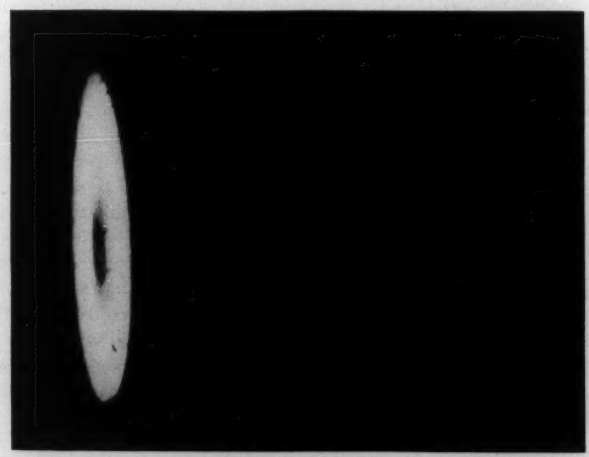
FIG. 15.



*a* *b*

Emission of X-rays from air in front of window of cathode-ray tube, as shown by X-ray pin-hole camera.

FIG. 16.



Emission of X-rays from various metals placed in front of window, as shown by X-ray pin-hole camera.

operated at 1 ma. and 200,000 volts and the exposure lasted fifty-four minutes. The different metals all received essentially the same electron bombardment and the pin-hole camera picture

FIG. 17.

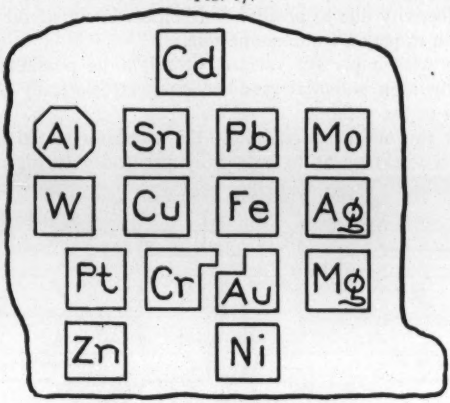



Diagram of metals used in Fig. 16.

shows the well-known differences in the efficiency of the different elements involved, as X-ray producers.

### (9) POSSIBILITY OF USING STILL HIGHER VOLTAGES.

(1) *With Tubes of the Present Type.*—The tube of Fig. 2, described in this article, is not well adapted to more than 250,000 volts, and could not be operated in air at appreciably more than 350,000 volts without arcing it over.

A larger model, with a 30.5-cm. bulb, an overall length of 150 cm., and larger and well-rounded electrodes, operates so well at 350,000 volts, which is as high as it has as yet been possible to test it, that it seems very probable that it can be made to operate at still higher voltages.

(2) *By Cascading Tubes.*—There is the interesting possibility, which must also be investigated, of putting several sections of cathode-ray tube in series, as shown in Fig. 18.

In this system the electrons would be raised to, let us say, a 350,000-volt velocity in the first section of the tube and then subjected to an additional 350,000 volts, perhaps, in each succeeding section.

The reasons why such a system may be necessary for the production of cathode rays of maximum attainable velocity are the following:

With a given degree of vacuum, there is a voltage at which the conductivity due to positive ion bombardment of the cathode may begin to play a troublesome rôle.

Even with a perfect vacuum, it might be possible, with a sufficiently high potential gradient, to electrostatically pull electrons out of the cathode.

With the multi-sectional tube the windows would be made of such a thickness as to be readily permeable to high-velocity

FIG. 18.

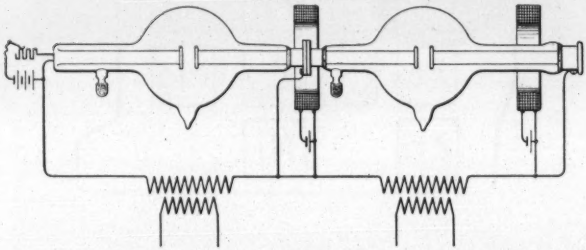


Diagram of cathode-ray tubes mounted in cascade for very high voltage.

electrons but not to positive ions. In this way, then, the cathode in each section would be subjected to bombardment by positive ions having no higher velocity than that corresponding to the voltage applied to the section in question.

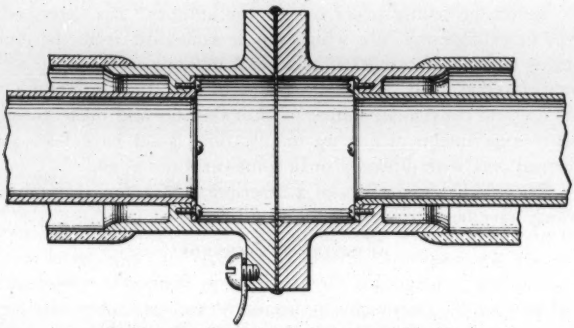
For a given total voltage, the use of several sections would also tend to lower the potential gradient at the surface of any one cathode.

In Fig. 18, the thin metal window between the two sections serves as anode of the left-hand section and at the same time as cathode of the right-hand section. As the space on both sides of it is evacuated, the thin metal foil does not require lateral support, and is simply soldered or welded in between the two heavy metal members, shown in detail in Fig. 19, to the outer ends of which the glass tubes are attached. The cathode end of the left-hand section and the anode end of the right-hand section would be essentially the same as the corresponding members of the tube shown in Fig. 2.

The solenoids indicated in Fig. 18 would prevent too much spreading of the cathode-ray beam and would be operated either from batteries, as shown, or from pulsating current in phase with the high-tension discharge.

It would not be necessary to limit the number of sections to two. Furthermore, each section could be a physically independent unit, in which case the section connected to the most highly negative end of the high-tension system could be like Fig. 2,

FIG. 19.



Detail of mounting of window between cascaded sections.

and all of the other sections could be, at both ends, like the anode end of Fig. 2.

It would be necessary to use metal shields at the cathode as well as the anode end of each section to prevent electrons, scattered in the metal window through which they entered the section, from getting to the glass.

The cascade system will be tried out as soon as the development of the single tube has been pushed to as high a voltage as possible.

In the following paper with Mr. C. N. Moore, certain physical, chemical and physiological effects of the cathode rays are described.

It gives me pleasure to acknowledge the able assistance which I have had in carrying out the work described in this paper, from Mr. Hubert Tanis, Jr., Mr. C. N. Moore, Mr. Leonard Dempster and Mr. George Hotaling.

# SOME EXPERIMENTS WITH HIGH-VOLTAGE CATHODE RAYS OUTSIDE OF THE GENERATING TUBE.

BY

W. D. COOLIDGE, Ph.D., and C. N. MOORE.

## (1) INTRODUCTION.

In the preceding paper one of the authors [1] has described a type of cathode-ray tube which can be sealed off from the pump and is capable of operation at several milliamperes and up to at least 350,000 volts (max.). Tubes of this type constitute an intense and convenient source of cathode rays and make possible many experiments which up to this time could have been performed only with difficulty or in some cases not at all.

The following consists of a description of a few experiments which have been tried with the new tube.

## (2) RAYING OF DIAMONDS.

Crookes [2] observed a discoloration in diamonds subjected to cathode-ray bombardment in his early vacuum tubes and says that some diamonds blacken in the course of a few minutes, while others require an hour or more to discolor. Moissan [3] tested the dark surface layer and concluded that it was graphite.

He later [4] studied the action of the rays from radium on diamond, and expresses his conclusion in the following words:

"The β-rays (electrons) effect a superficial darkening converting the surface into graphite in a manner similar to, but less strongly than, the more intense electrons in the cathode stream."

Lind and Bardwell,[5] from their experiments on the coloring of the diamond by radium radiation, conclude that diamond is not colored by the penetrating (beta and gamma) radiation coming from radium salts contained in glass tubes 0.5 to 1 mm. thick.

In the light of the above experiments it seemed interesting to subject diamonds to cathode rays from the tube. A current

[1] W. D. COOLIDGE, JOUR. FRANK. INST., this issue, p. 693.
[2] *Phil. Trans.*, Part II, p. 658 (1879).
[3] *C. R.*, **124**, p. 653 (1897).
[4] *Proc. Roy. Soc.*, **74**, pp. 47–49 (1905).
[5] JOUR. FRANK. INST., **196**, pp. 521–528 (1923).

of 1 ma., at 200,000 volts (max.), was used with an exposure time of thirty minutes at a distance of 1.3 cm. from the 0.0127-mm. thick nickel window.

Twenty-four diamonds [6] were rayed. These represented four different localities, Kimberly, Brazil, British Guiana and Australia, and a wide variety of colors.

To judge whether any color change had taken place, a method suggested by Doctor Kunz was employed. This consisted in finding, for each stone rayed, two other stones very near to it in color, and comparing it with these reference stones before and after raying.

All but two of the stones showed blue fluorescence. One of these two showed green and the other yellow fluorescence. None showed appreciable phosphorescence.

Not one of the diamonds showed any color change.

There is this difference between the early experiment of Crookes and the cathode-ray experiment which has just been described, that the former was carried out in a vacuum and the latter in the air. It would be interesting to repeat the Crookes experiment of raying the diamonds in a vacuum, but using a modern hot-cathode, high-vacuum tube and taking special precautions to guard against any metallic or other deposits on the diamonds, coming from the electrodes. Even to-day, under any but very good exhaust conditions, there is likely to be dark-colored deposit all over the inner walls of a vacuum tube, due either to spattering of the electrode material or to foreign matter coming out of the electrodes or, in case stop-cock grease has been used in the system, to hydrocarbon vapors.

### (3) RAYING OF FUSED QUARTZ.

Fig. 1 shows the ground and polished surface of a disc of fused quartz after exposing it for two minutes to cathode rays. The coloration is purple and disappears completely on strong heating before the Bunsen burner. On again raying the surface the purple coloration returns with perfect duplication of the original pattern.

By fracturing the disc and then raying the fragments, it was shown that the inhomogeneity extends throughout the mass and

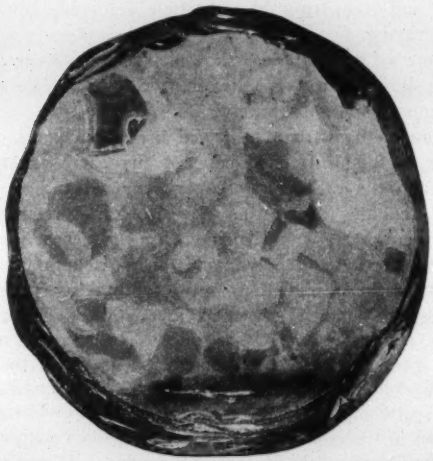
[6] These diamonds were obtained through the courtesy of Dr. George F. Kuntz.

is in no way connected with the grinding and polishing operations.

Quartz crystals from several different sources have been rayed and the only ones showing any color change are those from Herkimer County, N. Y., and these turn brown. The material used in making the fused quartz was from Brazil.

As the separate masses indicated in Fig. 1 are of the same general magnitude and shape as the broken crystal fragments which were put into the vacuum furnace to be melted down, it

FIG. 1.



Polished surface of fused quartz disc after exposure to cathode rays.

seems not unlikely that the picture shows the outline of the original masses, distorted by the melting operation.

Apparently chemical change has taken place in the furnace, putting some of this material into a condition where it will turn purple on exposure to cathode rays. The fact that all of the fragments are exposed to the atmosphere of the furnace and that some are brought to the condition where they will show a color change on raying, while others will not, together with the additional fact that the material appears in this respect to be so nearly uniform throughout the mass of the individual nodule, suggests the following as a possible explanation: The factor determining whether a

given crystal will or will not, after melting, change color on raying is to be found, perhaps, in a trace of impurity present originally in some crystals in such a form that it will not give the color change in the crystals, but chemically changed in the melted fragments to such a form that it will.

Further experiments will be tried to settle this point, for the full explanation may well be helpful in accounting for some of the peculiar luminescent effects and color changes which take place on raying various materials.

(4) X-RAY STUDY OF COLOR CHANGE PRODUCED IN HALITE (NaCl).

Half of a crystal of natural halite (NaCl) was covered with metal foil and the exposed half was rayed with 1 ma., at 200,000

FIG. 2.

Halite crystal with upper half exposed to cathode rays, and X-ray spectrograms of exposed and unexposed portions, each opposite corresponding portion of crystal.

volts for an hour, at a distance of 2.5 cm. from the window of the cathode-ray tube.

The crystal is shown at the left of Fig. 2, and at the right are two X-ray spectrograms made with a Bragg spectrometer, the upper from the exposed portion of the crystal and the lower from the adjoining unexposed portion. These spectra show no evidence of any change in the spacing of the atoms in the rayed portion of the crystal. This is in accord with the experience of Lind,[1] who rayed halite with β-rays and got no change in the X-ray spectrum.

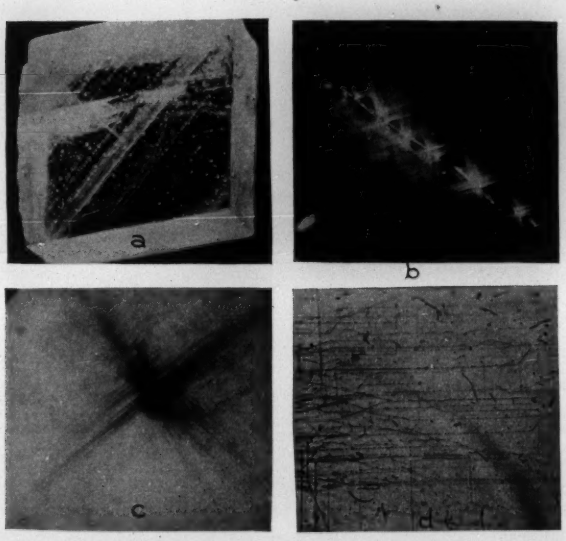
A similar experiment was tried on KCl; but in this case the material rayed was in the form of very tiny crystals, so small as to appear, after raying, uniformly darkened throughout their mass. They were then studied by the X-ray crystal powder method. Spectra made before and after raying appeared to be identical.

[1] JOUR. FRANK. INST., 196, p. 375 (1923).

### (5) CATHODE-RAY SCINTILLATIONS.[8]

Calcite crystals, upon being rayed, fluoresce strongly in the orange and remain highly luminous for several hours after raying. In addition to this they may show bright bluish-white scintillations. These have been observed while the crystal is undergoing bombardment and for as long as a minute after raying. By slightly scratching the rayed surface of the crystal

FIG. 3.




Traces of scintillations on calcite crystal. *a*, natural size; *b*, group of scintillations (x 75); *c*, one scintillation (x 200); *d*, section of one scintillation (x 360).

with any sharp instrument, the scintillations may be induced for as long as an hour after raying.

The area in the neighborhood of a scintillation loses all of its orange luminosity as the scintillation takes place and then appears dark against the bright orange background.

Under the microscope the spot where the scintillation took

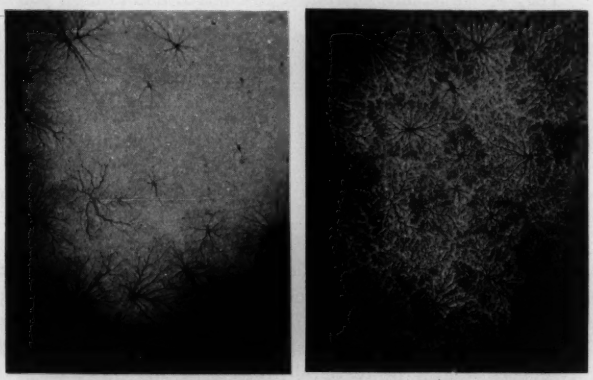
[8] This subject will be much more fully discussed in a paper which is now in course of preparation.

place is marked by a little crater with many tiny canals leading into it.

The whole canal system, except for the crater which breaks through to the surface, lies parallel to the surface and at a depth beneath it corresponding to, or less than, the range of the electrons in the calcite.

In Fig. 3, *a* shows a calcite crystal, in essentially natural size, on the upper face of which many scintillations have taken place; *b* shows the results of such scintillations, magnified 75 diameters;

FIG. 4.



*a* *b*

Traces of scintillations in celluloid, made at liquid-air temperature. *a*, one-second exposure; *b*, same area with ten seconds additional exposure.

*c* shows a single scintillation area magnified 200 diameters; and *d* shows individual canals produced by a single scintillation magnified 360 diameters and appearing as dotted straight lines.

Scintillations have not been observed in calcite at a temperature as high as 180° C. At liquid-air temperature they take place more readily than at room temperature.

Very few substances show scintillations at room temperature, but many substances show them at liquid-air temperature.

Fig. 4 shows two pictures of a piece of sheet celluloid, 0.178 mm. thick, which has been exposed at liquid-air temperature to cathode rays. The first, *a*, is after an exposure of one second,

and the second, *b*, is the same area after a further exposure of ten seconds. The canals which in the crystalline material, calcite, were straight, are here, in the colloidal material, seen to be curved.

Scintillations have been found to take place at liquid-air temperature in minerals, gums, organic acids, sodium salts of organic acids, alcohols, esters, other organic compounds and a few inorganic compounds.

The scintillations are electric discharges and are apparently to be observed only in substances which, at the temperature in question, are very good insulators.

They are apparently disruptive discharges due to high potential gradients which are built up in the surface layer of the dielectric by the electrons shot into it.

#### (6) DEPTH CHARGE IN VARIOUS INSULATORS.

Eguchi[9] has shown that if a thin layer of melted wax of suitable composition, such as a half and half mixture of rosin and carnauba wax, is allowed to solidify while in a strong electrostatic field, it becomes electrified throughout its depth and remains so, at least for years.

A similar effect is produced if cathode rays are shot at this same wax mixture while in a solid condition. It becomes charged to a depth corresponding at least roughly to the range of the electrons in the material.

The charge may be shown by the powder method of Lichtenberg, which is very fully discussed by Quincke.[10] It is helpful to use a stencil in raying as there is then a definite pattern to look for.

To remove any free surface charge, the wax may, after raying, be exposed to X-rays or to a flame. Very little care is needed in using the flame, as, unless the wax is actually melted in this operation, the depth charge remains.

Rosin and amber both show the same effect.

Calcite shows it, but only for a few hours after raying.

The same method applied to ordinary insulators such as hard rubber, glass and bakelite shows no permanent charge.

It is interesting to note that the only bodies which have been

[9] *Phil. Mag.*, **49**, pp. 178–192 (1925).
[10] *Ann. d. Phys.*, **32**, p. 91 (1910).

shown to take a permanent charge in this way are those bodies which show scintillations at room temperature.

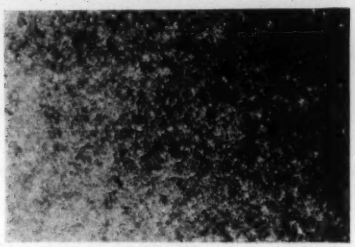
### (7) CATHODE-RAY SHADOW-PICTURES OF DELICATE TISSUES.

Cathode rays may be used to make pictures of thin delicate tissues which can be brought into intimate contact with a bare photographic film.

See Fig. 5, which is a cathode-ray picture of a piece of paper.

Close contact between paper and film was secured by means of a suitable frame, and the exposure was made at a distance of

FIG. 5.



Cathode-ray picture of piece of paper.

eight inches from the window of the cathode-ray tube, the latter being operated for 0.1 second with 1 ma., at 200,000 volts.

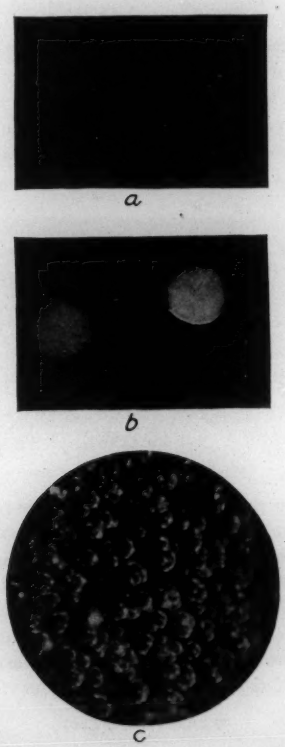
It would seem to be worth while to try making cathode-ray pictures of thin slices of organized tissue cut with the microtome, as very slight differences in density might in this way be shown, especially as the penetrating power can be made so low and can be so easily controlled. Structures not visible by ordinary light or by means of X-rays might possibly be shown in this way.

### (8) CHEMICAL EFFECTS.

In Fig. 6, *a* shows a piece of sheet glue which has been rayed over two small circular areas for a few seconds; *b* shows the same piece after slightly warming the glue; *c* shows one of the white areas of *b* magnified 100 diameters. The glue has broken down under the bombardment, and the gas formed blew bubbles as soon as the material was softened by heat.

The same thing happens with gelatin, shellac, cane sugar, rochelle salt crystals and celluloid.

FIG. 6.




Glue exposed over two small areas to cathode rays and subsequently heated. *a*, after exposure and before heating; *b*, after heating; *c*, portion of one of the exposed areas of *b* (x 100).

Aqueous solutions of cane sugar, starch and glycerine become acid to litmus on raying.

Milk, upon being rayed, quickly acquires a rancid flavor and strong odor. Butter behaves in exactly the same way.

#### (9) ACTION ON BACTERIA.

According to Dr. Hermann Strebel,[11] of Munich, cathode rays from a tube very similar to that of Lenard can be made to kill bacteria of all kinds.

Through the assistance of Dr. Ellis Kellert, Pathologist of the Ellis Hospital in Schenectady, it was possible to try the effect of the cathode rays from the new tube on bacteria. For these experiments the cultures were spread over the surface of a nutrient agar jelly, in some cases by flowing and in other cases by the help of a platinum wire. The following cultures were used: *Staphylococcus aureus, Bacillus coli, Bacillus prodigiosus,* and *Bacillus subtilis.* They were killed by an exposure of 0.1 ma., at 90,000 volts for one second, at a distance of 1 cm. from the window. The tube for these experiments was equipped with a 0.0127-mm. window of aluminum and was operated from high-voltage direct current at constant-potential.

Since the above experiments were tried, a paper on this subject has been published by Angerer, Pauli and Redwitz.[12]

#### (10) ACTION ON FRUIT FLIES, SNAILS AND COCKROACHES.

Fruit flies, upon being rayed for a small fraction of a second with 1 ma., at 150,000 volts, at a distance of 1¼ cm. from the window, instantly showed almost complete collapse and in a few hours were dead.

Water snails, exposed for very short periods of time, died either immediately or within a few days.

Cockroaches seemed badly injured when rayed for a quarter of a second and one was killed by being rayed for one-half second.

#### (11) EFFECT OF CATHODE RAYS ON PLANT TISSUES.

The effect of cathode rays upon plant tissues was studied by exposing portions of leaves of various plants to the rays, the remainder of the plant being protected by metal shields.

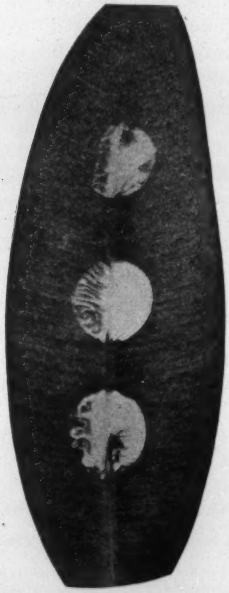
In one case, circular areas 2.5 centimetres in diameter on the leaf of a rubber plant (*Ficus elastica*) were exposed for ten seconds at a distance of 4 cm. from the window of the tube, which was operated at 1.0 milliampere and 200,000 volts. At the end of the exposure, the rayed areas were covered with white latex, Fig. 7, as though the cell walls had been punctured or in some other

[11] *Müncher med. Wochenschrift,* Jan. 20, 1914, p. 133.
[12] *Deutsche Ztschr. f. Chir.,* pp. 330–342, Oct., 1925.

way rendered permeable. The color of the exposed portions of the leaf gradually changed from green to brown and within about a week the rayed tissues throughout the thickness of the leaf had dried up.

Similar exposures were made on the leaves of geranium plants

FIG. 7.



Leaf of rubber plant after exposure of three areas to cathode rays.

with the same results except that no fluid appeared on the surface after the exposure.

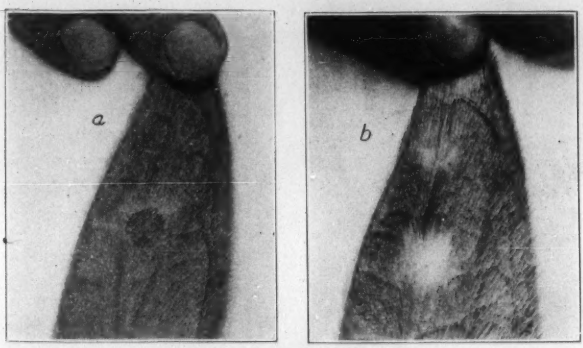
### (12) EFFECT OF CATHODE RAYS ON ANIMAL TISSUES.

The effect of cathode rays upon animal tissues was found to be as marked as that upon plant tissues. Small areas on the surface of the ears of rabbits were exposed to the rays. The effects produced varied from a slight tanning of the skin to complete destruction of the tissues throughout the entire thickness

of the ear, depending upon the time of the exposure and the amount of current used.

For example, two circular areas 1 centimetre in diameter on the ear of a gray rabbit, Fig. 8, were exposed for 0.1 of a second (the upper area) and 1.0 second, respectively, at a distance of 2.5 centimetres from the window of a tube operating at 1.0 milliampere and 200,000 volts. The rabbit was confined in a wire netting cage metallically connected to the anode end of the tube, and

FIG. 8.



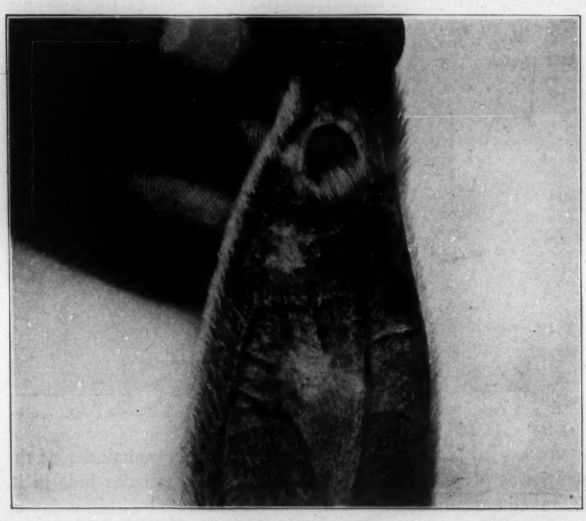

Rabbit's ear exposed locally for 0.1 second (upper area) and 1.0 second (lower area). *a*, one week and, *b*, seven weeks after exposure.

the unexposed portion of the ear, as well as the remainder of the rabbit, was protected by a metal sheet with a circular hole in it. The rayed areas immediately became inflamed and within a day or so were covered with scabs formed from an exudate coming from the rayed tissues. When the scabs fell off a week or two later, the hair came off with them and for a while bare skin remained. The hair follicles had not been destroyed, however, for about three weeks later, hair grew again. But some change had taken place in the hair follicles, for, on the area exposed for one second, the hair was snow white and eventually grew to be about twice as long as the surrounding gray hair. The hair that finally grew on the area exposed for 0.1 of a second consisted of some gray hairs and some white ones. In Fig. 8, *a* and *b* are photographs taken one week and seven weeks, respectively, after raying.

A third area, on the other ear of this same rabbit, was exposed for fifty seconds under similar conditions. In this case, a scab formed on both surfaces of the ear, and when the scabs eventually came off, a circular hole was left in the ear. White hair finally grew around the edge of the hole. (Fig. 9.)

A ten-second exposure over a larger area of the ear (2.5 × 5.0

FIG. 9.



Hole in rabbit's ear resulting from fifty-second exposure to cathode rays.

cm.) produced such destruction (Fig. 10) of the tissues that it finally seemed necessary to amputate the rayed portion.

All of the exposures so far described were made on the bare skin after removing the hair by shaving. The effect of the filtering action of the hair is roughly indicated by an experiment in which an area 2.5 centimetres square, on the side of the rabbit, was rayed through the hair. A ten-second exposure at 1 ma., and 200,000 volts, at a distance of 2.5 cm., gave essentially the same results as those obtained from the one-second exposure on the bare skin on the ear; a scab formed from exudate from the skin,

the hair came off with the scab, and, later, snow-white hair grew on the rayed area.

The effect of cathode rays on animal tissues would seem to offer a very promising subject for medical research. Much work has been done in the medical field with beta rays, but this has

FIG. 10.

Large area on rabbit's ear exposed to cathode rays for ten seconds.

been done with a mixture of beta rays, covering a wide range of velocities.

By operating the cathode-ray tube from a source of constant-potential continuous-current, it is now possible to experiment with cathode rays having a relatively narrow range of velocities. Furthermore, the maximum velocity is determined by the voltage used on the tube and is, therefore, controllable.

In conclusion, it is a pleasure for us to acknowledge the assistance which we have had in this work from Mr. Hubert Tanis, Jr.

**Separation of the Platinum Metals.**—S. C. OGBURN, JR., has made an extensive study of the reactions of the metals of the platinum group and has devised a procedure for their analytical separation (*Jour. Am. Chem. Soc.*, 1926, **48**, 2493–2512). The work was carried out at Washington and Lee University, and the University of North Carolina. To a solution, containing all the metals of the group as their chlorides or double chlorides and having a content of hydrochloric acid equal to 7 to 8 c.c. of the concentrated acid per 100 c.c., a 1 per cent. solution of dimethyl-glyoxime in alcohol is added at room temperature to precipitate the *palladium*. In the filtrate, *platinum* is precipitated by addition of a 2 per cent. solution of alpha-furil-dioxime in alcohol and boiling. The filtrate is concentrated, the oximes present are destroyed by means of hydrochloric acid and potassium chlorate, and *rhodium* is precipitated by addition of alcoholic solution of potassium nitrite while boiling, then permitting to stand over night. The filtrate is treated with concentrated hydrochloric acid, evaporated nearly to dryness, and then diluted; *ruthenium* is then precipitated by alcoholic solution of sodium hydroxide, and boiling. The filtrate is made distinctly acid with hydrochloric acid after removal of the alcohol by evaporation; iridium and osmium are then precipitated in a finely divided state by means of metallic zinc. The precipitate is treated with a freshly prepared solution of sodium hypochlorite which dissolves the osmium, and leaves the *iridium* unattacked. The solution containing osmium is rendered acid with hydrochloric acid, and the *osmium* is precipitated by use of zinc dust. Each precipitate is, of course, collected on a filter. The entire procedure may be used, in a slightly elaborated form, for the quantitative separation and determination of each metal of the group with an accuracy within 2.5 per cent. for each metal.

J. S. H.

**Test for Allantoin.**—WITHROW MORSE, of the Jefferson Medical College (*Proc. Soc. Exp. Biology and Med.*, 1926, **23**, 632–633), has devised the following qualitative test for the detection of allantoin, which is a ureide containing two urea residues and one glyoxylic acid residue. It is a product of purine metabolism. The allantoin may be isolated as allantoin silver nitrate, which is then suspended in water, or a portion of the original solution may be used in the test. A small flake of indol or five drops of 0.1 per cent. solution of indol in water are added and thoroughly mixed, then concentrated sulphuric acid is added so that it forms a layer beneath the aqueous liquid. If allantoin be present, it is hydrolyzed with the liberation of glyoxylic acid which reacts with the indol to form a colored ring similar to that obtained in the Hopkins-Cole reaction for tryptophane. This reaction is not given by uric acid (another product of purine metabolism), creatinine, or creatine.

J. S. H.

# MAGNETOSTRICTION.*

BY

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## INTRODUCTION.

BARTOL RESEARCH FOUNDATION. Communication No. 6.

IT IS customary in discussing experimental research in pure science to say something about the services which the work in question has rendered to the useful arts. This is hardly possible in the present instance, because, as far as can now be made out, no important application of the facts of magnetostriction—the change in dimensions which occurs when the magnetization of a body is altered—or of effects closely related to magnetostriction, has yet been made. There is, however, a connection between science and art of which this subject affords a good example. This is the dependence of advances in any chiefly experimental field upon the perfecting of instruments for measurement and upon the improvement of processes for the preparation of the materials to be examined. The principal periods of progress in magnetostriction have closely followed upon the invention of voltaic batteries capable of furnishing steady magnetizing currents, upon the design and construction of galvanometers for measuring such currents and the currents induced by changes in magnetization, upon every advance in the precision of length-measurement, and upon the development of a technique for the preparation and heat-treatment of homogeneous ferromagnetic alloys. The rate of progress thus secured has, to be sure, been very irregular during the more than eighty years that have elapsed since the first quantitative results were reported, and this irregularity has resulted in a good deal of uncertainty on the part of teachers and text-book writers as to the importance of the magnetostrictive effects in any complete discussion of ferromagnetism. While many physicists have stated that a theory of ferromagnetism, to have much weight, must

* A digest of material presented in three lectures delivered on the afternoons of Tuesday, April 13, Friday, April 16, and Wednesday evening, April 21, 1926, in the lecture hall at the Institute.

explain the various magnetostrictive effects, it has been the universal practice to build a theory without regard to these effects and then, by rather arbitrary additions, to give them place within it. One of the things here aimed at is the inclusion of magnetostrictive data in the foundation of a theory of ferromagnetism rather than in its superstructure.

A principal reason for the small attention hitherto paid to the subject lies in the opinion, held by nearly every one, that magnetostriction is a second-order effect. The experimenters themselves have been at fault here, for they have usually emphasized the difficulty of measurement rather than the significance of the results. It is quite true that a change in length of only a few parts in a million is something that most people can disregard without effort. The other aspect of the matter—about which much less has been said—is much more stimulating. No one would be likely to classify as secondary the fact that the magnetization of nickel in a moderate magnetic field can temporarily be reduced to one-tenth of its normal value by tension well within the elastic limit.

A contributing cause to the general apathy in regard to this topic is the idea that agreement regarding the facts of experiment is impossible. This seems to owe its origin to the very uncritical way in which the statements of the first experimenters have been swallowed whole by the later writers, even by those whose own experimental contact with the matters in question should have taught them better. It is easy to find contradictory statements in the literature of the subject, but a study of the differences in the materials used by the different observers, and in their methods of measurement, usually shows how the apparent contradiction has arisen, brings the earlier and later results into line, and suggests further interesting experiments. This is not the place for a critical review of the hundreds of contributions already available for such analyses. It must suffice here to give a short chronology of the subject, mentioning only a few of the men responsible for its progress.

### HISTORY.

A convenient point of departure is the observation by C. G. Page [1] at New Haven, in 1837, that a horseshoe magnet hung over a conductor wound spirally in a vertical plane emitted

a characteristic sound when a voltaic battery was connected to the ends of the spiral or disconnected therefrom. The sounds were due, although the experimenter did not know this, to the sudden changes in dimensions accompanying the changes in magnetization induced in the horseshoe magnet by the application and removal of the magnetic field of the spiral. Qualitative observations of like sounds were made in the succeeding decade by Delezenne,[2] Wertheim,[3] Beatson,[4] Marrian,[5] Matteucci,[6] de la Rive,[7] and others. These observers changed the form of the magnetizing coil, the material, shape, and mode of support of the sounding body. Notes on these sonorous effects have appeared at intervals ever since, but with the rise of more metric methods interest in them has largely diminished.

The quantitative era was ushered in by James Prescott Joule on February 16, 1842, in a lecture delivered at Manchester. His two papers on the subject, in 1842 and 1847, of which the second [8] contains an almost complete quotation of the first,[9] set a standard in the measurement of small linear displacements which was not bettered for many years, and the simple directness of his methods (every relevant variable being measured) leaves nothing to desire. He established the fact that iron (the only ferromagnetic metal he had) expanded along the axis of magnetization, and contracted at right angles thereto, in the strongest magnetic fields he could apply, the volume change in the same range of fields being inappreciable. He found the effect of tension was to increase the magnetization in a given applied field and to diminish the expansion due to magnetizing and made acute guesses as to the cause of certain peculiar effects in steel rods which would, if reported by his commentators, have been of value to many later experimenters.

After Joule's second paper there was a long period in which exact measurement was the exception rather than the rule. The changes in length due to magnetization were not investigated again for more than twenty-five years; the more easily observable effects of small strains upon magnetic properties were the subjects of numerous researches. In a long straight wire—a form of specimen especially convenient in magnetic studies—there are only two sorts of strain easily produced: Those due, respectively, to longitudinal pull along, and to torque about, a longitudinal axis. The former is homogeneous, *i.e.*, the strain in every ele-

ment of volume is the same. The latter is inhomogeneous, since elements of volume near the surface of the wire suffer greater distortion than those close to the axis. In spite of this complication the effects of torsion were studied first, probably because small twists are so much more easily measured than small linear expansions. Beatson,[4] in fact, observed an effect of twisting on the magnetization of iron wires as early as 1846. Six years later Wertheim[10] observed that when a magnetized wire was twisted there was a transient difference in electric potential between its ends.† Still later (in 1860) Wiedemann[11] published his first report on two effects which still bear his name. The direct Wiedemann effect, as it is called, is the twist produced in a wire, free at one end, which is simultaneously subjected to axial magnetic field and electric current. The inverse Wiedemann effect is the axial magnetization of a wire simultaneously subjected to axial torque and electric current.

The effects of tension upon magnetization were studied in detail by Villari.[12] The data he collected make it evident that in weak magnetic fields tension increases the magnetic permeability of soft iron, whereas in strong magnetic fields the effect is reversed (whence the term "Villari reversal"). Later researches show that in steel, in which Villari observed no reversal, the field value above which tension diminishes permeability, may drop nearly to zero.

The experimenters whose names have been mentioned, and a few others, published many notes and letters on their various effects, without adding very much to their first observations. It was not until Rowland[13] began (in 1873) to insist upon quantitative precision in magnetic studies that important progress resulted. Some of the experiments of Joule were repeated in 1873 by Mayer[14] at Stevens Institute. Mayer introduced improvements in technique as regarded length-measurement, but used less care than Joule in the measurement of other variables. His principal discovery—which might have been predicted from Villari's results, but was not—was that steel, after expansion along the axis of magnetization in weak fields, contracted so much in strong fields as to become shorter than at the start. He also, for the first time, made it clear that the length of a magnetized rod did not depend entirely upon the momentary value of the

† This effect was rediscovered by Matteucci in 1858.

applied magnetic field, but also depended upon whether the applied field had reached that momentary value by an increase from a smaller, or by a decrease from a larger value. This was a kind of hysteresis (the word "hysteresis" was not invented until eight years later [15]), although not the most important and interesting kind.

A repetition of some of Wiedemann's and Villari's experiments was undertaken by Sir William Thomson [16] (later Lord Kelvin) at Glasgow in the year 1875 and his results put on a firm metric basis some of the qualitative relations already known to exist. Perhaps the most striking things disclosed by these studies were the shift in the magnetic field for the Villari reversal as the applied tension was varied, the dependence of the magnitude of the effect in lower and higher fields upon the applied tensile stress, and the two-to-one correspondence of magnetization to twist in a fixed magnetic field when the magnetization was varied by cyclic changes in the twist. A new complication introduced at this time was the simultaneous application of tension, torsion, and magnetizing field.

About the year 1880 new names appear and rapidly attain prominence in the history of magnetostriction, but new types of experiment are few. To iron and steel, used in previous studies, Barrett [17] had already added nickel and cobalt, although the chemical purity and mechanical uniformity of specimens of the rarer ferromagnetic metals have only become at all satisfactory within the past few years. Bidwell,[18] Ewing [19] and Knott [20] were the sources not only of excellent experimental data of their own taking, but also of inspiration to those who have carried on the best work of the present century. Ewing, in particular, by his review of the subject in his "Magnetic Induction in Iron and Other Metals" (Electrician Series, 3rd edit., 1901), supplied a generation of experimenters with an adequate background of history and much stimulation to further endeavor. The influence of Knott upon his students at the Imperial College of Science in Tokyo resulted directly in the extensive researches of Nagaoka,[21] Honda [22] and others in Japan where the subject has remained alive to the present day. The article on "Magnetism" by Bidwell in the eleventh edition of the *Encyclopedia Britannica* is more recent and more complete in its treatment of magnetostriction than is Ewing's discussion of the same subject but contains a

pessimistic statement which may partially explain the reluctance of theoretical physicists to enter this field. Bidwell says: "The results [of studies on magnetostriction] embrace a multiplicity of details of which it is impossible to give an adequate summary." Despite this dictum it seems worth while to attempt some ordering of the experimental material. Before doing so, however, it will be of interest to see what theories have been offered in past years, and why they have been unfruitful.

### THE OLDER THEORIES.

J. J. Thomson,[23] in 1886, called attention to some important relations that might be expected to hold between the various magnetostrictive effects. The argument was dynamical and confined itself to what may be called reciprocal effects. Without attempting to say how the length of a wire is changed by changing its magnetization or how the magnetization of a wire in a fixed magnetic field is changed by changing its length, it is clear that these two effects must be interdependent. The analysis ignores irreversible processes—including magnetic hysteresis—and therefore the derived relations only hold with any exactness in cases where hysteresis is unimportant. Since, however, energy losses of all kinds cannot alter the sign of the final result of any small change in conditions, Thomson's theory is a safe guide to the algebraic sign of any effect when the sign of the reciprocal effect is known. For example, it says that in nickel, which always contracts when magnetized, tension must hinder and compression must favor magnetization along the axis of the applied stress. The theory has the advantage of generality and the disadvantage that it leaves out of account just the most interesting part of the subject, those hysteresis effects which must, if properly interpreted, tell much concerning the mechanism of ferromagnetism. From its dynamical character it would apply as well to the case of a hypothetical continuum as to the case of the atomic structure which modern methods have shown to exist in actual metals. For this reason it seems fair to refer to Thomson's analysis as the theory of continuous media.‡ Its later developments and refinements have not changed its inherent qualities nor made it in any sense a physical theory of magnetostriction, although this fact is sometimes overlooked.

‡ The somewhat earlier theory of Kirchhoff[24] is of the same type.

A much more physical picture of magnetostriction has been painted by S. R. Williams,[25] and others, on the canvas supplied by Ewing. Since Ewing's theory of ferromagnetism, whether that he presented in 1890 or the more modern version [26] of 1922, depends essentially upon the simultaneous orderly behavior of the individual elements in a region extensive on the atomic scale, the principal features of theories of magnetostriction on this basis may be summarized by calling them theories of manœuvre. These are incomparably more vivid than the theory of continuous media, and seem at first examination to leave little to desire. They attribute the changes in dimensions, which are the fundamental mystery of the matter, to gradual rotation, in unison, of non-spherical elements. In the first papers the elements were called molecules, but more recently it has become plain that they must be single atoms or parts of atoms. There is, however, a serious difficulty in the same place as before, for it appears that dissipation of energy is just as foreign to theories of manœuvre as to the theory of continuous media. It can be added, of course, but the theory gives no clue as to where the appropriate mechanism should be located nor in what it consists. The gross phenomenon has merely been scaled down to the element without resulting simplification, and whereas a solid metal is complex enough to leave within it room to explain hysteresis, the individual atoms, unless very different from the pictures drawn of them to explain other phenomena, have no suitable parts for the play of friction or the like non-conservative actions. It is worth remembering, too, that in the only case where it is reasonably certain that many atoms keep their magnetic axes parallel during a manœuvre—the case in which a magnetic field of large amount varies only in direction—hysteresis losses almost vanish.[27]

### CLASSIFICATION OF EFFECTS.

This digression on theories of magnetostriction sufficiently explains why more space is not devoted here to such theories. Before proceeding to the discussion of a recent theory of a rather different sort it will clarify the view of the experimental material to arrange a convenient system for classifying all the phenomena that ought to be covered by any competent theory. A suitable classification will make it evident that the number of really independent effects is very small. And first the problem will be

simplified, as it always is in practice, by limiting the forms of specimen to be considered to such as can be magnetized uniformly by simple means. It then appears that only the ellipsoid—of which the sphere forms a special case—and the annulus of negligible cross-section need to be dealt with. A special case of the annulus, and one of great importance, is found in the long cylinder (or prism), which may be regarded as part of an annulus of infinite radius. The long cylinder of elliptical (or circular) cross-section is also, of course, a special case of the ellipsoid obtained by making one axis infinitely long.

There are very few experimentally applicable agents which can act in the same way on every part of the specimens thus selected. In the ellipsoid, including the sphere, and the finite annulus, magnetic field and hydrostatic pressure are the only such agents. In the case of the long cylinder (or prism) homogeneous tensile stresses can also be produced, in the central portion of the length, by tractions applied at the ends of the specimen. In this special case also it is possible to cause an electric current to flow along the length of the piece so as to have a uniform distribution over the cross-section, though this does not, except in a still more special case, produce identical conditions at all points of the cross-section. In the case of a long *circular* cylinder only, torsional stresses of a calculable type may easily be applied, but this, too, in case the cylinder is solid, or a thick-walled shell, produces different conditions at different points. Finally, a long, thin circular cylindrical shell is exceptional in permitting uniformity of conditions within its cross-section as a result of axial electric current and axial torsion. This is the most special case it will be useful to consider.

Each of the five agents mentioned in the last paragraph may be associated with a measurable quantity dependent in principal degree upon only one of these agents, though affected more or less by the action of some or all of the others. Thus the magnetization, $I$, is dependent principally upon the magnetic field, $H$; the length, $L$, upon the tensile stress, $T$; the volume, $V$, upon the hydrostatic pressure, $P$; the fall of electric potential per unit length, $E$, upon the electric current density, $C$; the angle of twist per unit length, $A$, upon the shearing stress per unit area, $S$. Of the five pairs of quantities all but $V$, $P$ are vectors, and the homogeneity of the vectors within the specimens, assumed above,

sets certain limits upon their possible directions. In the sphere $H$ and $I$ may have any directions (they need not be parallel in the case of spheres cut from single crystals or from other anisotropic bodies). In the case of the annulus $H$ and $I$ must be in the plane of the annulus and perpendicular to its axis. In the long cylinder (or prism) $H$, $T$, $C$ and $E$ must have a common axis. In the thin circular cylindrical shell $H$, $I$, $T$, $C$, $E$, $S$ and $A$ may be the same for all points; $H$, $I$ and $T$ may have any directions in the tangent plane. In solid or thick-walled cylinders $\overline{I}$, $\overline{C}$, $\overline{S}$ and $\overline{A}$ (mean values) take the place of $I$, $C$, $S$ and $A$, since no one of these variables can, in such specimens, have the same value, other than zero, at all points of the cross-section. It is generally permissible to measure $L$ in any direction, though in some directions such measurement is generally much easier than in others. When an agent is changed in magnitude the change in any of the measurable quantities may be purely transient or it may persist as long as the agent is maintained at its new value. There is the usual complication to be expected if the changes in any variable are made cyclic.

It is now possible to write down all the possible combinations of the five agents, and each combination will correspond to a possible type of experiment. Further subdivisions may be made if, as is customary in any one experiment, only one of the agents is varied and only one of the measurables is directly observed. A suitable abbreviation in description is effected by writing first the letter chosen for the measurable quantity under consideration, next a dash, and finally the letters chosen for the agents to be varied. This symbolism lends itself readily to further refinement, the relative directions of non-parallel vectors being distinguishable by subscripts, the fixation of any variable by enclosing its letter in parentheses. For example, the symbol $\overline{I}—(HT)\overline{S}$ stands for the measurement of average magnetization (or changes in average magnetization) in a fixed field and under a fixed tension when a solid wire is twisted. A special case of great complexity, in which the average value of $T$ is zero, is presented in the bending of a rectangular prism [28]; for this the symbol $\overline{T}_\theta$ seems appropriate for the agent, $\overline{L}_0$, for the principal effect (the bending). Again $L_{(100)}—H_{(111)}$ naturally symbolizes the measurement of length (or of changes in length) parallel to a

tetragonal axis of a cubic crystal when a magnetic field is applied along a trigonal axis, the specimen, of course, being in the form of a sphere to ensure homogeneous magnetization.

Attention should be drawn to the fact that changes in elec-

TABLE I.

*Magnetostrictive and Related Effects.*

| | | | | |
|---|---|---|---|---|
| *I–H* (*a*) | *L–H* (*b*)<br>$L_y$–$H_x$ (*c*)<br>*L–T* (*a*)<br>*L–C* | *V–H*<br><br><br>*V–C* | *E–C* (*a*) | *A–S* (*a*) |
| *I–HT* (*d*) | *L–HT*<br>$L_o$–$HT_o$ (*e*) | *V–HT* | | |
| *I–HP* | *L–HP* | *V–HP* | | |
| *I–HC* | *L–HC* | *V–HC* | *E–HC* (*f*) | *A–HC* (*g*) |
| *I–HS* | *L–HS* | *V–HS* | $E_t$–*HS* (*h*) | *A–HS* |
| | *L–TC* | *V–TC* | *E–TC* (*i*) | |
| | *L–PC* | *V–PC* | *E–PC* | |
| *I–CS* (*j*) | *L–CS* | *V–CS* | *E–CS* | *A–CS* |
| *I–HTP* | *L–HTP* | *V–HTP* | | |
| *I–HTC* | *L–HTC* | *V–HTC* | *E–HTC* | *A–HTC* |
| *I–HTS* | *L–HTS* | *V–HTS* | $E_t$–*HTS* | *A–HTS* |
| *I–HPC* | *L–HPC* | *V–HPC* | *E–HPC* | *A–HPC* |
| *I–HPS* | *L–HPS* | *V–HPS* | $E_t$–*HPS* | *A–HPS* |
| *I–HCS* | *L–HCS* | *V–HCS* | *E–HCS* | *A–HCS* |
| | *L–TPC* | *V–TPC* | *E–TPC* | |
| *I–TCS* | *L–TCS* | *V–TCS* | *E–TCS* | *A–TCS* |
| *I–PCS* | *L–PCS* | *V–PCS* | *E–PCS* | *A–PCS* |
| *I–HTPC* | *L–HTPC* | *V–HTPC* | *E–HTPC* | *A–HTPC* |
| *I–HTPS* | *L–HTPS* | *V–HTPS* | $E_t$–*HTPS* | *A–HTPS* |
| *I–HTCS* | *L–HTCS* | *V–HTCS* | *E–HTCS* | *A–HTCS* |
| *I–HPCS* | *L–HPCS* | *V–HPCS* | *E–HPCS* | *A–HPCS* |
| *I–TPCS* | *L–TPCS* | *V–TPCS* | *E–TPCS* | *A–TPCS* |
| *I–HTPCS* | *L–HTPCS* | *V–HTPCS* | *E–HTPCS* | *A–HTPCS* |

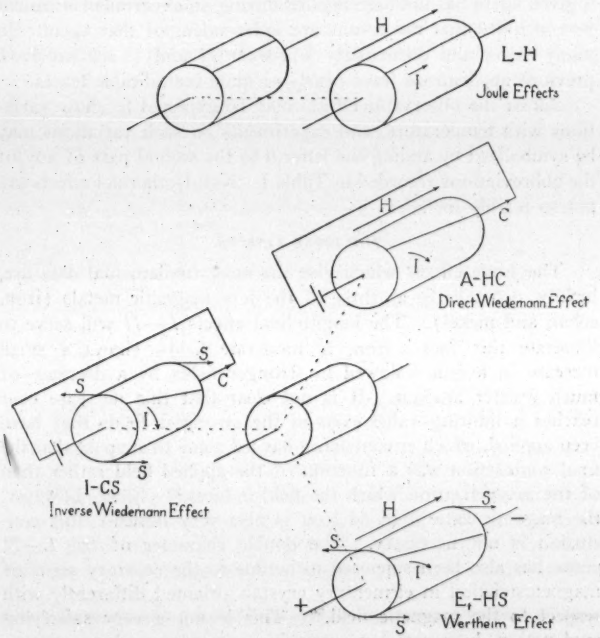
(*a*) Included for theoretical reasons only because the existence of magnetostriction must modify the shape of *I–H*, *L–T*, *E–C*, and *A–S* curves.
(*b*) Longitudinal Joule effect.
(*c*) Transverse Joule effect.
(*d*) Inverse longitudinal Joule effect (which may show Villari reversal) is the difference between *I–HT* and *I–H*.
(*e*) Guillemin effect on elasticity in flexure.
(*f*) Magneto-resistance effect.
(*g*) Direct Wiedemann effect.
(*h*) Wertheim effect.
(*i*) Elasto-resistance effect.
(*j*) Inverse Wiedemann effect.

trical conductivity, while included in this scheme, appear in it only as changes in the fall of potential, $E$, along a cylindrical or prismatic specimen carrying a current of density $C$. There are cases in which $E$ only differs from zero during the time in which

the steady state is being established; such transient effects will be designated by writing $E_t$ for $E$.

Table I contains a complete list of the typical experiments that can properly be included under magnetostriction and closely

FIG. 1.




Schematic diagram illustrating the relations between the principal magnetostrictive effects. The more complex effects may all be explained either by changes in lengths along and across the direction of the magnetization (Joule effects) or by changes in the amount and direction of the magnetization due to changes in dimensions however produced (inverse Joule effects). (*Cf.* Table I.)

related effects. Many of the possible combinations, *e.g.*, *V*—*P*, are omitted because they depend only upon elastic properties and are not affected in any known manner by the ferromagnetic properties of the specimen. Many of the included combinations are so complicated that no investigator would willingly undertake

the corresponding experiments, but some of these combinations have inadvertently been applied and they are listed, not in the hope that more work will be done on any of them, but to show their relations to simpler cases (Fig. 1) from which more useful information can be derived. It should be noted that just because a given agent has not been applied during an experiment it cannot be concluded that the results are independent of that agent. In many cases, and particularly whenever $H$ and $C$ are involved, previous applications leave nearly or quite ineradicable traces.

All of the observable effects may be expected to show variations with temperature, and experiments on such variations may be symbolized by adding the letter $\theta$ to the second part of any of the abbreviations recorded in Table I. Non-isothermal effects are not so readily included.

### THE JOULE EFFECTS.

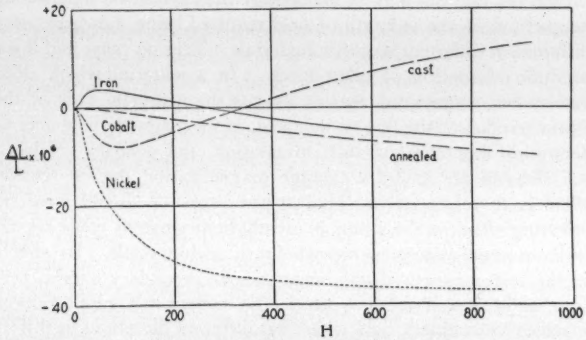
The Joule effects which give the most fundamental data are, luckily, not alike in all three of the ferromagnetic metals (iron, cobalt and nickel). The longitudinal effects $L$—$H$ will serve to illustrate this fact. Iron, in moderate fields, shows a small increase in length followed in stronger fields by a decrease of much greater amount. It is not clear that this decrease ever reaches a limiting value even in the strongest fields that have been applied, which circumstance has led some to suppose that the final contraction was a function of the applied field rather than of the magnetization which the field induces.[29] Since, however, the magnetic saturation of iron is also very difficult, this conclusion is not necessary. The double character of the $L$—$H$ curve has also been supposed to be due to the contrary signs of magnetostriction in elementary crystals oriented differently with respect to the magnetic field.[30] This is not a very satisfying explanation, however, for successive expansion and contraction also occur in a single crystal of iron when the applied field is parallel to a digonal axis.[30]

Nickel, in all fields which affect its length at all, contracts, and the contraction, unlike that in iron, seems to reach a limit in attainable fields. The limiting contraction is relatively large, amounting to nearly 4 parts in 100,000. It should be mentioned that iron and nickel have different cubic crystal structures at room temperature, the former being body-centred, the latter, face-

centred. Conditions in the face-centred structure appear to be much the simpler.

Cobalt has been tested in a relatively pure state by but few observers. Its behavior is very different, depending upon its previous history. Since this metal has two crystal structures, face-centred cubic and hexagonal close-packed, either or both of which may be found in a specimen at room temperature, such complication is not surprising. There is some evidence that the

FIG. 2.




Joule effects in iron, cobalt ("as cast" and "annealed"), and nickel, after K. Honda and S. Shimizu (*Phil. Mag.*, (6) **6**, 392-400 (1903)).

Joule effect in cobalt is opposite to that in iron, the metal contracting in moderate fields and expanding in very strong fields. Some unpublished work by H. A. Pidgeon [31] suggests, however, that the second stage may be almost completely suppressed by careful annealing. (Fig. 2.)

The Joule effect in Heusler alloys (a limited range of alloys containing manganese, copper and a metal from the third, fourth or fifth group in the periodic table), which are ferromagnetic without containing iron, cobalt or nickel, is an expansion up to the highest fields yet used.[32] It is a fair speculation that the ferromagnetic constituent in these bronzes is manganese and that if manganese alone could be made ferromagnetic (as has once been reported [33]) it would be opposite to nickel in the sign of its magnetostriction. The four consecutive elements Mn(25), Fe(26), Co(27), Ni(28) would then exhibit a symmetrical set

of Joule effects ranging from simple expansion to simple contraction with both signs of change—in the two possible sequences —in the intervening members of the series.

### ATOMIC ORIENTATION.

A great simplification is at once effected by referring all the various magnetostrictive effects to a common cause, the orientation § of the magnetic axes of the atoms. The parallel orientation, in one sense, of all the atomic axes corresponds to magnetic saturation, and can only be effected by the application of a strong magnetic field, the strength of field required being, however, very different in different magnetic materials. Tension may, and does, produce orientation of other kinds. In a material which elongates when magnetized, tension orients the magnetic axes of the atoms parallel to the line of pull, but, if no magnetic field acts to determine the sense of such orientation, the polarity produced is indeterminate and the average magnetization due to tension alone is therefore zero. Hydrostatic pressure should have no orienting effect on the atoms in a truly homogeneous cubic crystal or in an aggregate composed wholly of such crystals. Its effects on the ferromagnetic metals must then be secondary except, perhaps, on hexagonal cobalt. An electric current in a wire produces a transverse magnetic field which has different directions at different points within the cross-section, and torsion produces tensions and compressions which vary both in direction and in amount. Either of the last-named agents, therefore, should, when acting alone, produce characteristic distributions of orientation within a specimen of suitably symmetrical form.

The orientation of any atomic axis being uniquely determined by two independent coördinates, the problem of magnetostriction can now be stated. It is, in fact, the evaluation of these two coördinates for each atom of the specimen when subjected to any number of coöperating agents. The problem in this form is just as insoluble as before, but some progress can now be made, for average values of the desired coördinates throughout the specimen are furnished by measurements of changes in magnetization, length and twist, and averages of another kind by meas-

§ This very awkward word seems unavoidable. Alignment will not do because other types of regularity must be included, and suitable names for these are intolerable (*e.g.*, helicination, applanation, circumlineation).

urement of changes in electrical resistivity ($E/C$). Volume changes (except perhaps in hexagonal cobalt) are, as has already been said, to be regarded as secondary. They probably measure the degree of inhomogeneity in orientation rather than the algebraic or geometric sum of quantities to which a value can be assigned for each atom. Experiments on volume changes have, as might be expected on these premises, yielded very discordant results and in the best work only small dilatations (positive or negative) have been observed.

### CRYSTALS.

The theories of manœuvre insist that easy magnetization can only be effected when each atom is as symmetrically situated as possible, *i.e.*, when the crystal in which it finds itself is free from foreign elements, local defects in crystal structure and irregularities of all sorts. The interest thus directed to the study of single ferromagnetic crystals, first by Pierre Weiss [34] and his pupils, and more recently by C. W. Heaps,[35] W. L. Webster [36] and W. E. Ruder,[37] has already disclosed some surprising facts. The most striking result is that the magnetic properties of cubic crystals of iron and magnetite, unlike the mechanical properties of all cubic crystals in the elastic range, are not even approximately isotropic. Of all the magnetic properties, too, the magnetostriction is the least nearly isotropic, $L—H$, for example, having different signs for different directions in a sphere cut from magnetite (Heaps) and for rods cut in different orientations from large iron crystals (Webster [30]). A less striking but no less important fact is that magnetic hardness may coexist with the single-crystal state. The proponents of theories of manœuvre avoid the apparent contradiction by supposing that the regions in which the atoms manœuvre together are small with respect to the gross crystal structure while remaining large with respect to the single atom. This *ad hoc* assumption emasculates these theories considerably.

The magnetic anisotropy of crystals is most clearly shown by the fact that in them the magnetization is not parallel to the applied field except when the field is applied along one of a few especially symmetrical directions. Weiss [38] has introduced a concept in this connection which deserves mention although it is somewhat confusing to most people. He ascribes the non-

parallelism of $I$ and $H$ to the action of what he calls a "molecular field," $M$, called into being by magnetization and proportional thereto, but so directed that the vector sum of $M$ and $H$ is parallel to the observed direction of $I$. The value of $M$ has in some crystals to be set so high that any attainable value of $H$ is negligible in comparison. In other cases—particularly in iron crystals, as worked out by W. L. Webster[36]—$M$ and $H$ are of the same order at half saturation. No physical explanation of the "molecular field" which satisfies even the author of the term has yet been devised, and the essential artificiality of the concept is abundantly apparent. It seems fair to say that it is merely an analytical substitute for the interatomic forces and torques which act upon those atoms in a crystal whose magnetic axes are not parallel to the applied magnetic field.

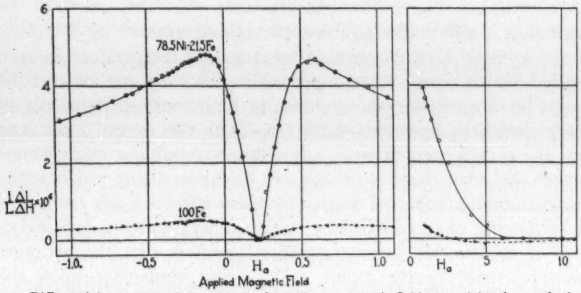
### ALLOYS.

Besides reasonably pure iron, cobalt and nickel, a fair selection of carbon and other steels, a few of the Heusler alloys, and magnetite ($Fe_3O_4$), a couple of binary alloy systems have been studied for magnetostrictive effects. The binary alloys are those of iron with cobalt[39] and with nickel,[39, 40] respectively. A great deal more work has been done on the second of these alloy systems than on the first. The high-nickel region of the iron-nickel series is especially interesting because here, at about 81 per cent. nickel and 19 per cent. iron, the magnetostriction passes through zero for all values of the applied field, alloys with less nickel expanding (Fig. 3), and alloys with more nickel contracting. X-ray methods have shown that over a very wide range in composition—from pure nickel to an alloy with only 30 per cent. nickel—including this region, the two elements form true solid solutions, the crystal structure of the solutions being similar to that of nickel but with steadily increasing mean distance between atom-centres as the iron content increases.[41] The alloys in this general region have been given the name "permalloy" because under proper conditions their magnetic permeabilities greatly exceed those of previously known materials.[42]

In a material which has zero Joule effect, the inverse Joule effect, $I—HT$, should be indeterminate. For a small positive Joule effect, such as that in permalloy with 78 per cent. nickel, the inverse Joule effect should be enormous, and positive, *i.e.*,

tension should increase the magnetization at constant field. For a small negative Joule effect, such as that in permalloy with 84 per cent. nickel, the inverse Joule effect should also be enormous but, in this case, negative. It was, in fact, by measurements upon the inverse Joule effect that the composition of permalloy for zero magnetostriction was first determined.[43] Direct measurements of increase or decrease in length under an applied magnetic

FIG. 3.




Differential magnetostriction (slope of length *vs.* magnetic field curves) by the method of Hobbie (*Phys. Rev.*, (2) **10**, 456-466 (1922)). Data by L. W. McKeehan and P. P. Cioffi (*Phys. Rev.*, (2) **21**, 707 (1923)). The curves are symmetrical about $H_a$=0.23 gauss because an applied field of this amount was necessary to compensate the component of the earth's magnetic field along the axis of the sample. Note especially that these curves are tangent to the $H_a$-axis at true field zero, and that the curve for iron, by crossing the $H_a$-axis, indicates a maximum extension with subsequent contraction. In permalloy with 78.5 Ni—21.5 Fe there is no sign of such contraction even at considerably higher applied fields than those here shown.

field (with or without tension) have since been made to check this finding.[40] The refinements of technique introduced to permit exact plotting of the $L$—$H$ curve when, as in some of these alloys, the maximum change in length to be measured may be less than one part in a million, have permitted a decision to be reached as to the way in which magnetostriction in pure metals and in alloys begins and proceeds.

### ATOMIC MAGNETOSTRICTION.

It has already been mentioned that existing theories of ferromagnetism are not very clear on the subject of magnetostriction. When it was observed that none of them even hinted at the possibility of finding a material, like permalloy, much more easily magnetized than pure iron, something really had to be done about it. The result has been the development of a new way

of looking at magnetostriction.[44] All the peculiarities of permalloy are easily explained from the new viewpoint, and when the same reasoning is applied to the behavior of a pure metal the general problem of ferromagnetism seems much nearer to a solution. It will be convenient to invert the historical order of this theoretical development and to describe first not how it accounts for permalloy, but how it regards the magnetization of a pure metal consisting, as properly annealed specimens do, of large crystals with the least possible internal strains. And at the beginning it will suffice to think of a single crystal.

In a nickel crystal annealed from a high temperature in zero applied field it is natural to suppose that the magnetic axes of the atoms lie in a considerable number of directions—perhaps not in every direction, however—with respect to the crystal axes, and that the net magnetization in any volume containing many atoms is zero, *i.e.*, that there is no favored direction along which a disproportionate number of atomic magnets point. Each atom in a nickel crystal is surrounded by twelve others and the very existence of measurable magnetostriction suggests that the arrangement of these twelve atoms will depend somewhat upon the direction of the magnetic axis of the atom at the centre of the group. Admitting this, and extending the picture, it is seen that while the atom-centres lie on the average at the points of a geometrically perfect space-lattice, in this case face-centred cubic, any selected atom will probably be displaced slightly from its point on such a space-lattice. Such a displacement corresponds to the different interatomic forces between the atom in question and its various neighbors. Each atom is in equilibrium, but the stability of orientation is clearly different for different atoms. The stability may be very great if a change in orientation of the atom considered would require large displacements of the neighboring atoms, *i.e.*, if "atomic" magnetostriction is important.

Let now a magnetic field be applied by external means. This raises the stability of orientations nearly parallel to its line of action, and lowers the stability of other orientations. As the field strength is gradually increased the orientation of single atoms at separated points within the crystal will become unstable and sudden changes in orientation will occur. The correspondingly sudden changes in the (at least) twelve interatomic forces which act on the adjacent atoms will set up elastic vibrations to be

transmitted through the crystal and converted into heat. This dissipated energy is suitable for the explanation of hysteresis.|| As in all atomic events, it is convenient to assume that unless the change in potential energy for a possible change in conditions (here a change in orientation of the atomic axis with respect to the applied field) is sufficient to supply the energy which must be dissipated in the accompanying events, the change will not occur.

The behavior of the metal atoms has been pictured as if they stood still in the absence of applied magnetic fields. Actually, of course, they are supposed to be executing random motions about their positions of equilibrium, the energy of such motions depending upon the temperature. At temperatures sufficiently below the Curie point,¶ however, it is experimentally evident that such random motions do not often result in changes of atomic orientation, for if they did the specific heat would be higher than it is in the lower range of temperatures, and permanent magnetization would be impossible even in closed magnetic circuits.

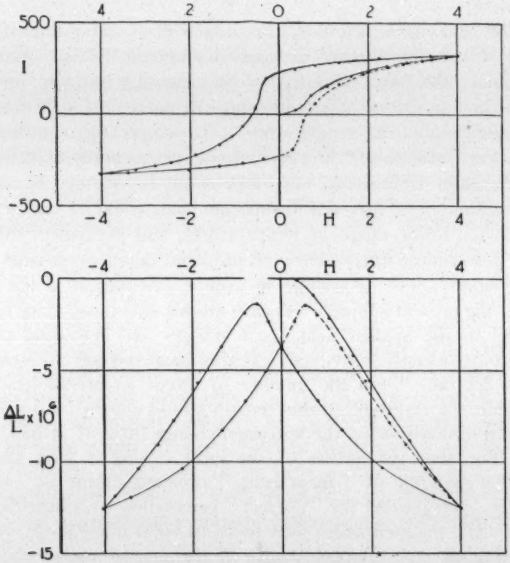
Another effect of change in atomic orientation is the very gist of the present subject. As each atomic axis is set more nearly parallel to the applied field, local stresses are developed or, to put it more exactly, previously existing local stresses are oriented in the crystal. Until the number of atoms so oriented is considerable, the resultant strains remain almost purely local and the external dimensions of the specimen change little, if at all. It is clear, too, that the nature of the local strains is such that at first the stability of "incorrectly" oriented atoms is, on the average, increased by the "correct" orientation of a neighboring atom. The magnetization thus tends to build itself up by events occurring at non-adjacent points in the crystal rather than by spreading from the points first affected, and the average stability of unoriented atoms at first increases. As magnetization proceeds, however, a time will come when the local strains begin to overlap and the local stresses to diminish. This permits a rapid increase of magnetization and a relatively still more rapid change

|| Of course if the magnetic moment of each atom is spatially quantized with respect to the resultant of the local and applied magnetic fields, these sudden changes will involve the emission of low-frequency radiation, the energy of which will be part of that lost in hysteresis.

¶ The temperature above which ferromagnetic characteristics disappear.

in external dimensions, *i.e.*, in observable or "gross" magnetostriction. The nature of this gross magnetostriction should be a safe clue to the nature of the atomic magnetostriction to which it is due. It is just as magnetostriction begins to show itself that large scale instability may occur, and many atoms may change their orientation simultaneously, or nearly so, giving evidence of

FIG. 4.




Magnetostriction in nickel. Data by L. W. McKeehan and P. P. Cioffi. Note hysteresis and failure of return to original length.

such changes by the Barkhausen effect, the audible effect of transient currents in neighboring electric circuits.[45]

In the saturated crystal the strain, to be sure, is a maximum, but the stress is zero again, not merely on the average, but everywhere. It might be thought that when the external field is removed under these circumstances the saturated state would be retained. Actually, however, the local fields near the free surfaces are very different in direction from those in the interior

of the piece and when these local fields are left to act alone there is partial reversion to the original state of chaos, with partial demagnetization. But even when the average magnetization is brought to zero by putting on a reverse field, or, better yet, by reducing a large alternating field slowly to zero, the final state of the nickel crystal will be different from that at the beginning, for more atomic axes will remain nearly parallel to the axis of first magnetization than to any other line. The proof that this is so is found in two facts: The $I$—$H$ curve for a second magnetization is different from that for the first; the $L$—$H$ curve is not only different, but the length for zero magnetization is permanently reduced. (Fig. 4.)

In a specimen containing many crystals there will be a further averaging process, but the principal stages of the process of magnetization in a single-crystal specimen will still occur.

In iron or in cobalt the process will be the same as in nickel, although the type of local stresses due to partial magnetization, and the final state of strain (magnetostriction) will be different in each metal. In low fields (but up to relatively high magnetizations) the magnetostrictions of iron and nickel are opposite in kind, the former expanding along the line of magnetization, the latter contracting. The transverse effects are also opposite in the same range of applied field. It seems fair to suppose that the atomic magnetostrictions are also complementary. Quantitative comparison is hard because the crystal structures—and especially the closeness of packing of the atoms—are different in the two pure metals.

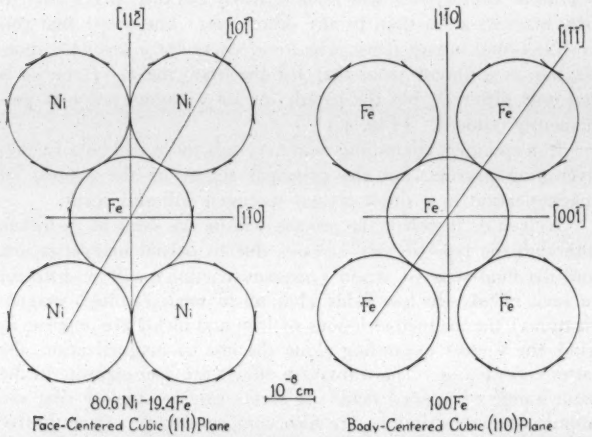
In a solid-solution alloy the two sorts of atoms are under the same conditions and the comparison becomes easier.

### PERMALLOY.

The process of magnetization is really simpler in permalloy than that in pure iron or in pure nickel. In a properly proportioned alloy the very easy magnetization makes it reasonable to suppose that the local stresses due to the orientation of iron atoms can be very nearly annulled by the orientation of adjacent nickel atoms. All that is necessary is that a small group of iron and nickel atoms shall be capable of simultaneous orientation by the applied field without much change in the interatomic forces between any member of the group and any non-member. The

energy which must be dissipated when the atoms forming the group are oriented by the applied field is thus reduced to such a value that the small supply of energy due to the original non-parallelism of the atomic axes and a weak applied field is sufficient to pay the tax that will be demanded. The group coöperates in

FIG. 5.




Schematic diagram illustrating a possible grouping of four nickel and one iron atom in permalloy of critical composition, and, to the same scale, the most nearly similar group of five iron atoms in pure iron. The plane of the group is in each case a crystal plane on which atoms are most thickly set. The atoms are represented in each case as spheres of diameter equal to the mean distance between closest pairs of atom-centres as determined by X-ray crystal analysis.

the change and the contribution to magnetic hysteresis losses is reduced almost to nothing.

This, in brief, is the "explanation" of permalloy. The properties of permalloy are now to be viewed as a direct consequence of the cancellation of magnetostriction, and it is important to note that the cancellation must be effected in small groups of adjacent atoms. It would be easy enough to join long rods of iron and nickel of such relative lengths that the net longitudinal Joule effect, $L—H$, of the composite bar would be zero, but this would not at all reduce hysteresis or make magnetization easy. The compensation must be due to a very intimate mixture of the two metals and should, *ceteris paribus*, be better the more perfect

the mixing. Thorough working and annealing at temperatures where diffusion tends to improve homogeneity are, in the absence of favoring stresses, essential to the best quality.

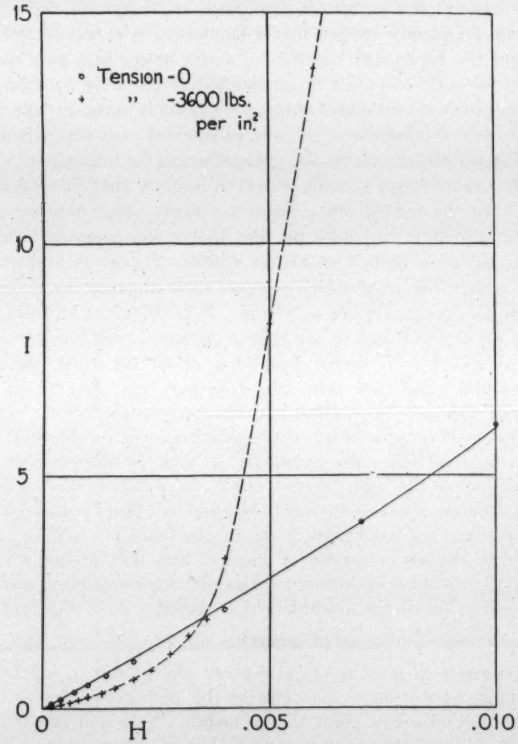
The idea that a "compensated" group must contain very few atoms has two bases. In the first place, if the groups were large it would frequently happen that a haphazard selection of groups, just like the haphazard selection of single atoms in a pure metal, during the early stages of magnetization would leave considerable numbers of iron and nickel atoms so isolated between groups with already oriented members that the subsequent easy magnetization of these residual atoms in like groups would be impossible. The low hysteresis losses actually observed indicate that this does not occur. In the second place, there are many other physical phenomena which fit well with the idea that a few atoms can rather readily act as a unit in processes wherein energy is emitted or transformed, but none which suggest such capacity for coöperation by any great number of atoms. It is therefore encouraging in the present instance to see that a group of only five atoms—four of nickel and one of iron—has about the right chemical composition (80.6 per cent. Ni, 19.4 per cent. Fe) to fit the observed critical composition for zero magnetostriction. (Fig. 5.) There is no possibility of arranging groups of this size in a way consistent with cubic symmetry so that the atom-centres are the points of the face-centred cubic space-lattice disclosed by X-ray studies. It must therefore be supposed that if such groups of five atoms are associated in the magnetizing process the association is entirely a matter of chance, and that atoms will be differently grouped in different cycles of magnetization, especially if the direction of the applied field is varied.

### STRESS.

The effect of tension (or any other stress) not so intense as to produce plastic flow, is to change the interatomic forces in a definite way at every point in the metal. This will change the stability of different orientations whether or not an applied magnetic field is also acting. In nickel, for instance, tension will increase the stability of orientations with the atomic magnetic axis at right angles to the axis of tension and decrease the stability of orientation with these axes parallel. Of course, if the positive stability of any orientation is low and if it is reduced

to a negative value by tension, the orientation must change to one of positive stability just as well when tension is being applied as when changes in applied magnetic field are responsible for the

FIG. 6.




Effect of tension on initial magnetization of permalloy with 78.5 Ni—21.5 Fe. Note depression of initial permeability and great increase in permeability at slightly higher field values.

changes in stability. There is this difference, however, that whereas a magnetic field may be applied in either sense along its line of action there is no such distinction in the case of a tensile stress. The magnetic field, therefore, when it makes any orien-

tation untenable, determines at the same time which way the magnetic moment shall be directed in the new orientation; the tensile stress, on the other hand, is as likely to turn the magnetic axis into one of two possible directions which are equally inclined to the axis of stress, as into the other. The orientations developed by a magnetic field are thus always accompanied by magnetization in the ordinary sense, *i.e.*, a change in the magnetic moment per unit of volume. The orientations developed by tension *alone* are not so accompanied and should wholly escape purely magnetic measurements.

Orientation of atoms by tension is not, however, without magnetic effect. For one thing, the subsequent application of a magnetic field will disclose very different states of affairs if tension has or has not been applied. (Fig. 6.) Orientations by tension should also give rise to transient electromotive forces in associated circuits of the same general type as those observed by Barkhausen in magnetization of the ordinary sort.

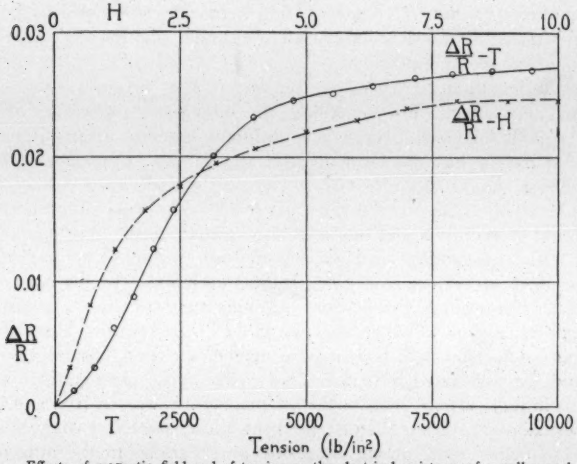
The orientation of atoms by tension will have its clearest result, however, in modifying magnetostriction. In the case of nickel the resultant orientation of atomic magnetic axes across the axis of tension, which reduces the ease of magnetization along the axis of tension, will increase the magnetostriction (contraction) and the associated hysteresis losses over those characteristic of the initially chaotic distribution of magnetic axes. In iron and in permalloys with less than 81 per cent. nickel the effect of tension is to orient some atomic magnetic axes parallel to the axis of tension. Subsequent magnetization will be easier, and this for two reasons. In the first place, the stability of all oblique orientations has been diminished by tension and may now be upset by smaller applied fields. In the second place, the atoms which have been oriented by tension alone already have their magnetic axes lying along the line of the applied magnetic field, but about half of them have a polarity contrary to that which the field favors. These atoms can contribute to magnetization by reversing their polarity without otherwise changing their orientation and, therefore, without contributing to magnetostriction, that is to say, without dissipating much energy. This means that they will be so reversed, with rise of magnetization in the specimen as a whole, by relatively weak applied magnetic fields. The magnetostriction (expansion) of iron will therefore be reduced by apply-

ing and maintaining tension; the hysteresis loss will be diminished by the same means.

### RESISTIVITY.

Besides using changes in $I$ and $L$ to indicate changes in atomic orientation, the experimenter may use for the same purpose changes in a third quantity which depends upon the orientation of

FIG. 7.




Effects of magnetic field and of tension on the electrical resistance of a well-annealed permalloy wire with 78.5 Ni—21.5 Fe. Note different aspects of the two curves, and approximate agreement in maximum changes. There is a small increase under tension due to changes in length and cross-section so that the maximum changes in resistivity are more nearly equal than the changes in total resistance.

atoms and which can therefore be affected by applying either magnetic field or tensile stress. This third quantity is the electrical resistivity, $E/C$. How the orientation of the atomic magnetic axes changes the resistivity is still a complete mystery, but that it does so is sufficiently plain from the behavior of permalloy in this respect. It has been shown [40, 43] that in permalloy with less than 81 per cent. nickel tension reduces the magnetostrictive expansion and increases the magnetic permeability, while in permalloy with more than 81 per cent. nickel tension increases the magnetostrictive contraction and decreases the magnetic permeability. To this must now be added that in

permalloy with less than 81 per cent. nickel tension increases electrical resistivity,[46] that in permalloy with more than 81 per cent. nickel tension decreases electrical resistivity, and that magnetization increases the resistivity in both regions of composition. (Fig. 7.) Moreover, just as combined tension and magnetic field produce in a 78 per cent. nickel alloy no greater limiting magnetization than does a strong magnetic field alone, so tension and magnetic field together produce in the same material no greater limiting change in resistivity than does a strong magnetic field alone.[46] Changes in electrical resistivity, like magnetostriction, but unlike changes in magnetization, do not depend upon the agreement in one sense of the orientations of the atomic axes. It is therefore possible in this case, as it was not in the case of magnetic measurements alone, to detect directly the orienting effect of tension, and to show that by tension alone as much increase in resistivity can be effected as by magnetization alone. This completes the proof that the two agents act in the same way upon the same elements. Since the only way we can suppose a magnetic field to act is in orienting atomic magnetic axes, this must also be a mode of action of tensile stress. How the two agents may do this has been suggested in the preceding paragraphs.

### MECHANICAL PROPERTIES.

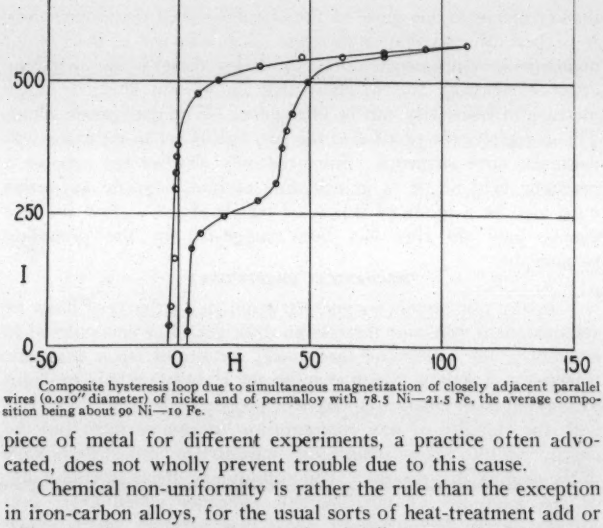
Having put atomic magnetostriction in as the very basis of ferromagnetic behavior there is no difficulty to be encountered in explaining the effects of mechanical conditions upon magnetic behavior. Anything which changes the rigidity or bulk modulus of a metal must change its magnetic properties because it changes both the stability of any configuration of atomic axes and the energy put into elastic strains and impulsive actions at each stage in the magnetizing process. The well-known effects of cold-work, of the additions of non-magnetic elements which enter into solution in the ferromagnetic metal, and of other agents which affect the properties loosely included under the term "hardness," are thus easily understood.

### NON-UNIFORMITY.

So far an effort has been made to restrict the argument to cases in which magnetic fields, stresses and strains, except in regions of atomic dimensions, are sensibly uniform in direction and amount. The effects of torsion, bending and electric current

flow in solid wires have formed exceptions, but these have not been discussed in any detail. The possibilities which arise when this restriction is removed are so numerous that only a few typical kinds of non-uniformity can even be mentioned within the limits of this presentation. The prevalence of non-uniformity in actual specimens makes the comparison of different magnetostrictive effects very difficult and unsatisfactory. Even use of the same

FIG. 8.



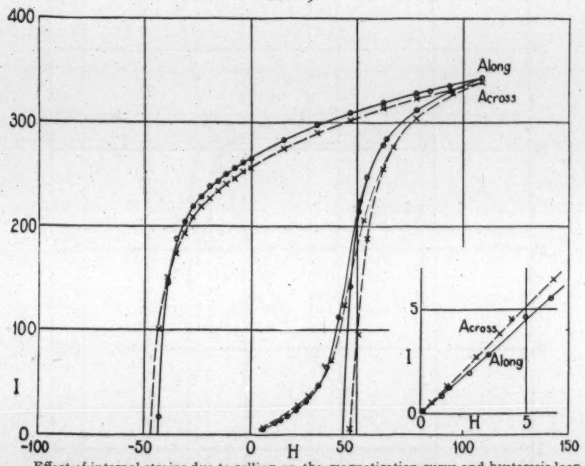

Composite hysteresis loop due to simultaneous magnetization of closely adjacent parallel wires (0.010" diameter) of nickel and of permalloy with 78.5 Ni—21.5 Fe, the average composition being about 90 Ni—10 Fe.

piece of metal for different experiments, a practice often advocated, does not wholly prevent trouble due to this cause.

Chemical non-uniformity is rather the rule than the exception in iron-carbon alloys, for the usual sorts of heat-treatment add or subtract carbon and produce a skin of different composition from the interior. If the piece has one long dimension, and if it is subsequently magnetized parallel to this dimension, this kind of non-uniformity is similar in its magnetic effects to that produced by laying alongside each other wires of different materials. (Fig. 8.) The magnetization is different in the different layers, sometimes even different in sign, and there is a very characteristic distortion of the magnetic hysteresis loop by which non-

uniformity of this type is easily recognized.[47] Loops distorted by this cause are narrow where they cross the $H$-axis and relatively wide at high positive and negative magnetizations. The true remanence is high and the coercive force may be nearly as low as that of the magnetically softest streaks in the piece, whether these happen to form the surface layer or the central core, especially if the soft material occupies half or more of the

FIG. 9.




Effect of internal strains due to rolling on the magnetization curve and hysteresis loop of hard-rolled nickel sheet (rolled from 0.0075" to 0.001" without annealing). Strips cut "along" and "across" the rolling direction were measured by the magnetometer of Bozorth (*J. Opt. Soc. Amer.*, **10**, 591-598 (1925)). The former shows greater inhomogeneity of internal strains than the latter. After annealing at so low a temperature that preferred orientation of crystals still persisted there was no detectable difference in the magnetic behavior of the two strips.

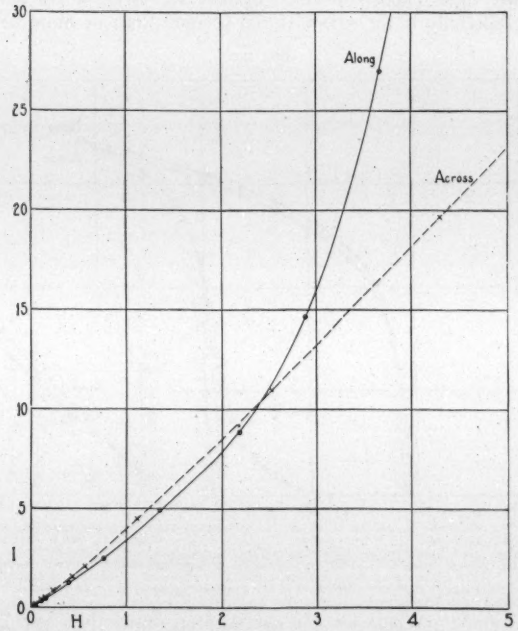
total volume. Whenever a hysteresis loop shows these peculiarities it is a safe deduction that the magnetic flux is unevenly distributed over the cross-section and a reason for such a state of things may confidently be sought.

### INTERNAL STRAIN.

Internal strains are even more frequent sources of this type of non-uniformity than are differences in chemical composition. In cold-worked metal, of course, the incidence of internal strains is quite unavoidable, and they are non-uniform because their

geometric sum over the whole cross-section must, when external applied forces are absent, always be zero. Since tension and compression change magnetic properties—an inverse Joule effect—the stretched and compressed parts of a cold-worked sheet or

FIG. 10.



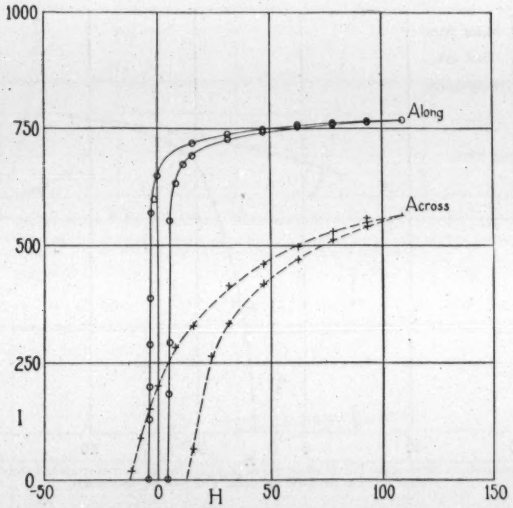

Effect of internal strains due to rolling on the initial magnetization curves of hard-rolled permalloy with 78.5 Ni—21.5 Fe. Preparation, method of measurement, preferred orientation of crystals, and effect of annealing similar to samples for which curves are given in Fig. 9.

wire are different magnetic materials in a very real sense, and their combination shows the same sort of magnetic behavior as combinations of chemically different metals. (Figs. 9, 10, 11.)

Intense internal stresses, in parallel, can also be maintained by holding straight a suitably bent wire. The hysteresis loop may in such a case become discontinuous, an appreciable part of the

whole change in magnetization taking place without change in applied field when atoms in a region which is held saturated by local strain begin to reverse their magnetic axes. Examples of such discontinuities in nickel have recently been reported, but without explanation, by Forrer.[48] (Fig. 12.) The discontinuity occurs when the compressed nickel reverses its magnetization. In

FIG. 11.



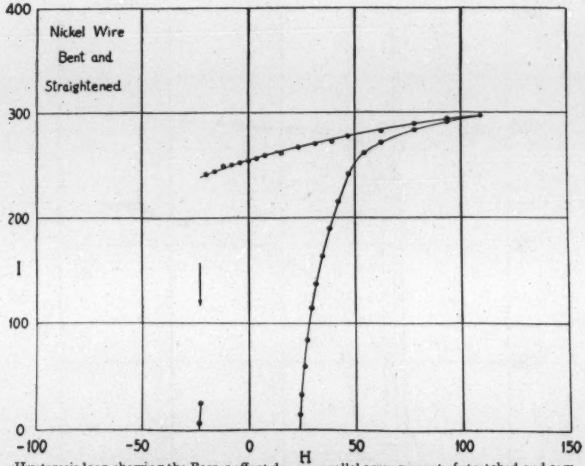

Effect of internal strains due to rolling on the hysteresis loops of hard-rolled permalloy with 78.5 Ni—21.5 Fe. Same specimens as in Fig. 10.

iron the Villari reversal in the effect of tension makes the effect of unbending less simple, since it is not possible to orient all the atoms in iron parallel to a single line by either tension or compression.

It is not so obvious that the rapid cooling of metal objects, even when changes in crystal structure and separation of dissolved impurities are not involved, produce internal stresses quite large enough to modify magnetic properties in the same way. The great internal stresses which exist in rapidly cooled glass are familiar facts, however, and stresses of the same sort will be set

up in any body which has been heated to a temperature at which it is at all plastic and which has then been quickly cooled to a temperature at which it is rigid. The explanation lies in the fact that the internal stresses due to temperature gradients in the hot, cooling, body are continually being reduced by plastic flow until the temperature of the surface passes below the minimum tem-

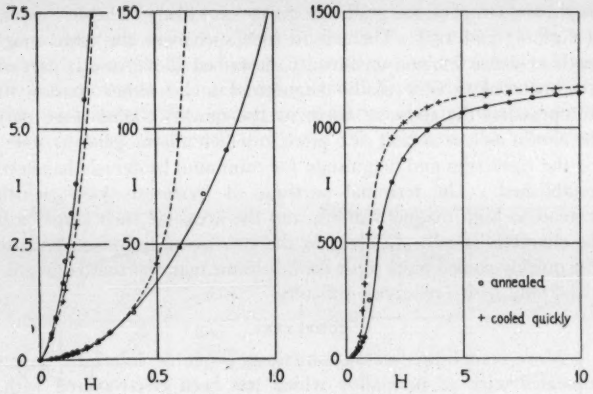
FIG. 12.




Hysteresis loop showing the Forrer effect due to parallel arrangement of stretched and compressed fibres in hard-drawn nickel wire (0.010″ diameter).

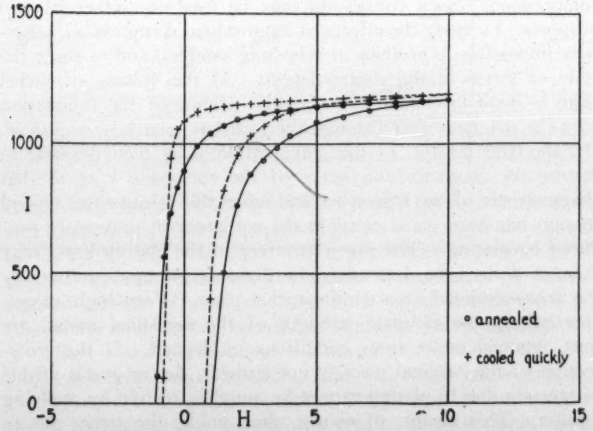
perature for notable flow. At this moment there are still temperature gradients in the piece, the centre being hotter than the surface. In cooling to room temperature the centre therefore shrinks more, in proportion, than does the surface, and if the piece does not split apart on cooling to room temperature it is only because the central portions remain in tension, the surface portions in compression. The actual distribution of stresses depends upon the shape and dimensions of the piece and upon the temperature differences which existed within it at the moment when the coolest parts became non-plastic. These temperature differences depend most directly upon the rate of cooling. In especially strain-sensitive materials the effects of the internal

FIG. 13.




Effect of internal strains due to rapid cooling (from below the α-γ transformation temperature) on the magnetization curve of an Armco iron ring.

FIG. 14.




Effect of internal strains due to rapid cooling (from below the α-γ transformation temperature) on the hysteresis loop of an Armco iron ring. Same specimen as in Fig. 13.

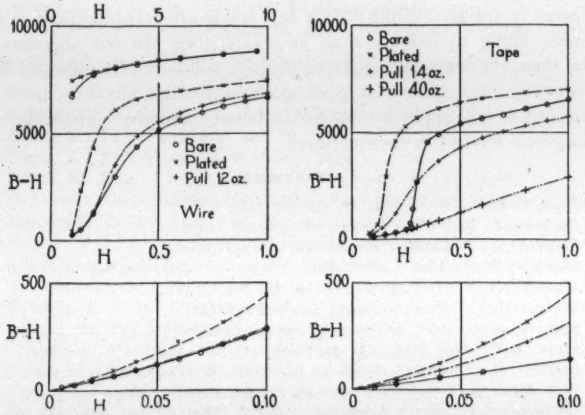
strains thus produced in thin specimens—in which cooling may be rapid and temperature gradients great—are likely to be enormous. (Figs. 13 and 14.) There is in such specimens the usual magnetic evidence for non-uniformity, described above, and if part of the material is very easily magnetized under either tension or compression the coercive force of the quickly-cooled piece may be almost as low as that of a piece in which a homogeneous stress of the right sign and magnitude for minimum hysteresis has been established. The terminal portions of hysteresis loops which extend to high magnetizations, and the areas of such loops, will be characteristically different in the two cases, however, because the quickly-cooled piece must contain some material that is magnetized only with (relative) difficulty.

### COMPLEXES.

More complicated cases sometimes prove of interest. In an annealed wire of permalloy which has been electroplated with, say, nickel, the wire and its coating have at the same time different chemical compositions and different longitudinal stresses, the nickel plate being in tension, the foundation nickel-iron alloy in compression. Such composites may be used for either of two purposes: To study the effects of longitudinal compression, otherwise impossible to produce in very long samples, and to study the state of stress in the electro-deposit. If the volume of nickel plate is small in comparison with the volume of the foundation wire the magnetic flux through the nickel is relatively negligible. By applying tension to the combination it is then possible to restore the magnetization curve of the composite wire to that characteristic of the bare wire, and when this is done the applied tension has been made equal to the compression previously produced by plating. The previous stress in the plating itself may then, if desired, be determined by dividing the applied force by the cross-sectional area of the nickel plate. Very high values, approaching the ultimate strength of the deposited metal, are thus obtained under some conditions of deposit. If the cross-section of the original piece is not circular, the magnetic effects of stresses due to plating cannot be annulled merely by applying tension. This means, of course, that, unlike the stress due to forces at the ends of the specimen, the stresses due to plating are not in this case homogeneous. (Fig. 15.)

Non-uniformities of other sorts may easily be imagined and rather frequently occur in practice. For example, suppose that two magnetic materials, with different characteristics in bulk, are mixed in a single alloy but not in a single solid solution. Here the adjacent grains of the metal have different magnetic properties, the magnetic flux, except at saturation, follows zig-zag paths (of minimum total reluctance) and the magnetic characteristics

FIG. 15.




Effect of strains due to nickel-plating upon the magnetization curves of wire and tape of permalloy with 85 Ni—15 Fe. The strains so produced in wire are homogeneous and can be annulled by pulling the plated wire; the strains in tape are not homogeneous and cannot be so annulled.

of the alloy should be some sort of average between those of the two components. The hysteresis loops for such aggregates should be very different from those for a parallel arrangement of the various materials involved, and should be indicative of the true state of affairs. The strongly curved loops of magnet-steels and of severely cold-worked metals are typical of this condition.

There is one more magnetic peculiarity of non-uniform pieces which deserves special mention. The magnetostrictive strains in the regions first magnetized produce alteration of stress in regions yet to be magnetized, so that the results in such pieces should differ somewhat from those due to the magnetization of like arrangements of identical materials which are not coherent parts

of a single metallic structure. Since the greatest magnetostrictive strains are not as large as the limiting elastic strains which can easily be produced in ordinary metals, this mutual elastic action of the parts is generally a matter of secondary importance.

## CONCLUSION.

The study of magnetostriction, long supposed to be interesting but not important, has recently flared up again in a way that promises to throw light on the obscure subject of interatomic forces in solids. While only a few solids are ferromagnetic it is encouraging to observe that, as solids, they are not abnormal. If, then, the interplay of atoms in these solids can be followed by magnetic means there is good probability that any conclusions arrived at will apply as well to solids not themselves amenable to magnetic control or measurement.

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UPON entering on his duties as non-resident lecturer in chemistry at Cornell University, Professor Panneth, of the University of Berlin, delivered a public lecture on "Ancient and Modern Alchemy." The speaker might well claim to be an exponent of the modern phase of the art. In referring to his recent work in conjunction with his assistant, Doctor Peters, that seems to have accomplished the transmutation of hydrogen into helium, he says, "I am compelled to confess to you that I was not only interested in the literature of alchemy, but I even ventured to take up experiments along this line. For some reasons, particularly because of the sharpness with which the smallest quantities of helium can be recognized, I thought it promising to test the transmutability of hydrogen into helium and the paper mentioned above (Berichte of the German Chemical Society) contains the results of experiments which we have carried on during the last three years. This transmutation we accomplished not by electric discharges, as tried by others, but by simple catalytic

action. I hope further to be able here at Cornell to show that these experiments work as well in the new world as in the old."

As reported in *Nature* (Oct. 9, 1926), they perfected a spectroscopic method by which as small a quantity of helium as $10^{-8}$ to $10^{-9}$ c.c., that is, $10^{-12}$ to $10^{-13}$ gm., could be detected. Thus was shown the production of helium by a mixture of thorium B and thorium C. In addition this method has led to the discovery of a natural gas in Germany that contains .19 per cent. by volume of helium, somewhat more than half as much as is found in the Canadian natural gas from which helium has been derived in considerable quantities.

Attempts were made to convert hydrogen into helium by subjecting it to the silent electric discharge in an ozone tube and, in another experiment, to a strong discharge in a Geissler tube, but in both cases no success was attained. No better results were had by bombarding certain salts with cathode rays, according to Lord Rayleigh's suggestion. Then as much as a litre of hydrogen was passed through heated palladium. The result was promising but not above suspicion, since the strength of the helium lines obtained bore no proportionality to the quantity of hydrogen used. At last a successful treatment was come upon. Finely divided palladium absorbed hydrogen at room temperature. Oxygen was admitted and the hydrogen burned at the surface of the catalyst, palladium. After getting rid of the resulting water vapor and excess oxygen, the gas left was conveyed to a narrow capillary tube, where a spark passed through it. When the palladium had been in contact with hydrogen for twelve hours, the spectrum of the residual gas displayed several helium lines and, furthermore, the longer the time of contact, the greater the quantity of helium manifesting itself by the discharge. Not all preparations of absorbing palladium showed the same activity, but all grew less effective with use. Heating, whether in oxygen, in hydrogen, in a mixture of these gases or merely in a vacuum, restored at least in part the power to develop helium. In no case was helium obtained from palladium preparations that absorbed no hydrogen, while, on the other hand, there are preparations that absorb the gas and yet produce no helium. Palladium preparations that had stood a long time were found to contain helium. Less active than palladium are finely divided platinum and pyrophoric nickel.

A formidable list of possible errors was presented to the minds of the investigators. They guarded against them all and hold that the production of helium from hydrogen has been accomplished.

About .28 calorie of energy should have been liberated in their experiments, but so small a quantity would be hard to detect and it is not surprising that they failed to trace it. They incline, moreover, to the opinion that this energy would appear as Milligan rays or gamma rays rather than as heat. G. F. S.

# MAXWELL'S EQUATIONS AND ATOMIC DYNAMICS.

BY

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MECHANICS.

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IN THE solution of the problems of mechanics and electrodynamics, the considerations involved are considerably simplified when the equations can be expressed in an invariant form as regards transformation of coördinates.

Let us consider first the case of a particle of unit mass, possessing $n$ degrees of freedom, then the differential equations of motion in the Lagrangian form can be written,

$$\frac{d}{ds}\left(\frac{\partial L}{\partial \dot{q}^i}\right) - \frac{\partial L}{\partial q^i} = 0 \qquad i = 1 \cdots n, \tag{1.1}$$

where

$$\dot{q}_j = \frac{dq_j}{ds} = \text{velocity in } q_j \text{ direction.}$$

Now let us further restrict the motion to the case where the Lagrangian function $L$ can be expressed as a quadratic function of the velocities

$$L = g_{\alpha\beta}\dot{q}^\alpha \dot{q}^\beta$$

summed on $\alpha$ and $\beta$ from 1 to $n$.*

In the further development of this analysis, we shall suppose that all repeated Greek indices are summed over this range.

Then the equations of motion become

$$2g_{\alpha i}\frac{d^2q^\alpha}{ds^2} - \frac{\partial g_{\alpha\beta}}{\partial q^i}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds} + 2\frac{\partial g_{\alpha i}}{\partial q^\beta}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds} = 0,$$

which we can write in the following form as a matter of notation only,

$$g_{\alpha i}\frac{d^2q^\alpha}{ds^2} + \Gamma_{\alpha\beta,i}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds} = 0,$$

* The $\alpha$'s, $\beta$'s, 2's, 3's, etc., appearing throughout this paper are used as superscripts and not as exponents.

where

$$\Gamma_{\alpha\beta,i} = \frac{1}{2}\left(\frac{\partial g_{\alpha i}}{\partial q^\beta} + \frac{\partial g_{\beta i}}{\partial q^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial q^i}\right). \tag{1.2}$$

If we define a set of functions $g^{ij}$ by means of the relationship

$$g^{\alpha i} g_{\alpha j} = \delta^i_j. \qquad \delta^i_j = \begin{matrix} 0 & i \neq j \\ 1 & i = j \end{matrix},$$

then solving these equations by means of determinants, we can express them in the form

$$\frac{d^2 q^i}{ds^2} + \Gamma^i_{\alpha\beta}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds} = 0, \tag{1.3}$$

where

$$\Gamma^i_{j\kappa} = \Gamma_{j\kappa,\lambda} g^{\lambda i},$$

the $g_{ij}$ being supposed symmetrical:

$$g_{ij} = g_{ji}.$$

Let us next consider the change in the form of the differential equations (1.3) when we make a change in the coördinates of a type defined by the relation

$$q^i = q^i(x^1, x^2, \cdots\cdots x^n), \tag{1.4}$$

where $q^i$ is a function only of the $n$ new coördinates

$$x^1, x^2 \cdots\cdots\cdots x^n.$$

Then

$$\frac{dq^i}{ds} = \frac{\partial q^i}{\partial x^\alpha}\cdot\frac{dx^\alpha}{ds}$$

and

$$\frac{d^2 q^i}{ds^2} = \frac{\partial^2 q^i}{\partial x^\alpha \partial x^\beta}\cdot\frac{dx^\alpha}{ds}\cdot\frac{dx^\beta}{ds} + \frac{\partial q^i}{\partial x^\alpha}\frac{d^2 x^\alpha}{ds^2},$$

so that (1.3) becomes

$$\frac{\partial q^i}{\partial x^\alpha}\frac{d^2 x^\alpha}{ds^2} + \frac{\partial^2 q^i}{\partial x^\alpha \partial x^\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} + \Gamma^i_{\alpha\beta}\frac{\partial q^\alpha}{\partial x^\mu}\frac{\partial q^\beta}{\partial x^\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0,$$

the $\Gamma$'s now being defined in terms of the $x$'s.

If we now define a new function $\dot{\Gamma}^{i}_{ij\kappa}$ by

$$\dot{\Gamma}^{\alpha}_{ij}\frac{\partial q^\kappa}{\partial x^\alpha} = \frac{\partial^2 q^\kappa}{\partial x^i \partial x^j} + \Gamma^\kappa_{\alpha\beta}\frac{\partial q^\alpha}{\partial x^i}\frac{\partial q^\beta}{\partial x^j}, \tag{1.5}$$

then (1.3) takes the form

$$\frac{d^2 x^i}{ds^2} + \dot{\Gamma}^i_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0,$$

the determinant of the transformation being different from zero.

The form of the differential equation is now invariant under the transformation (1.4), provided the $\Gamma$'s are regarded as transformed according to the relation (1.5).

We could also have arrived at these results by a consideration of the mode of transformation of the $g$'s.

The first integral of the equations of motion (1.3) is

$$g_{\alpha\beta}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds} = \text{constant} \tag{1.6}$$

which must be invariant under the transformation since the trajectories of the particle are independent of the particular set of coördinates used to define them.

The first integral of the equations of motion may be derived directly from Lagrange's equations by observing that in the particular case where the Lagrangian is a quadradic function of the velocities, *i.e.*, the Lagrangian and the energy function are identical, the constancy of the energy integral is equivalent to the statement that

$$L = \text{constant};$$

applying the transformation to the $g$'s, we obtain

$$g_{\alpha\beta}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds} = g_{\alpha\beta}\frac{\partial q^\alpha}{\partial x^\mu}\frac{\partial q^\beta}{\partial x^\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$$

$$= g'_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = \text{constant},$$

so that the $g$'s are transformed according to the law

$$g'_{ij} = g_{\alpha\beta}\frac{\partial q^\alpha}{\partial x^i}\frac{\partial q^\beta}{\partial x^j}. \tag{1.7}$$

Substituting this result into the expression for the $\Gamma$'s, we arrive again at the relationship (1.5); the previous method, however, for the determination of the transformation of the $\Gamma$'s has a more general application since it is independent of the existence of the first integral of equations (1.3).

From this result we see that we could have simplified our calculations, provided we had defined a new operation called covariant differentiation such that the covariant derivative of $\frac{dq^i}{ds}$ with respect to $q^j$ $\left(\frac{dq^i}{ds}\right)_j$ is given by the relation

$$\left(\frac{dq^i}{ds}\right)_j = \frac{\partial}{\partial q_j}\left(\frac{dq^i}{ds}\right) + \Gamma^i_{j\alpha}\frac{dq^\alpha}{ds},$$

or
$$\left(\frac{dq^i}{ds}\right)_\beta \frac{dq^\beta}{ds} = \frac{d^2q^i}{ds^2} + \Gamma^i_{\alpha\beta}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds},$$

for then the equation (1.3) takes the form

$$\left(\frac{dq^i}{ds}\right)_\alpha \frac{dq^\alpha}{ds} = 0, \tag{1.8}$$

which is independent of the form of the coördinate system used to define the trajectories found as solutions of (1.6).

Let us next consider a more general equation of motion which is also invariant under a transformation of coördinates.

$$\left(\frac{dq^i}{ds}\right)_\alpha \frac{dq^\alpha}{ds} = \frac{dq^i}{ds} g_{\mu\nu}\frac{dq^\mu}{ds}\frac{dq^\nu}{ds}, \tag{1.9}$$

then under the transformation (1.4) the equation (1.9) becomes

$$\frac{\partial q^i}{\partial x^\alpha}\left(\frac{dx^\alpha}{ds}\right)_\beta \frac{dx^\beta}{ds} = \frac{\partial q^i}{\partial x^\alpha}\frac{dx^\alpha}{ds} g_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds},$$

since $g_{\mu\nu}\frac{dq^\mu}{ds}\frac{dq^\nu}{ds}$ is invariant for this transformation or

$$\left(\frac{dx^i}{ds}\right)_\alpha \frac{dx^\alpha}{ds} = \frac{dx^i}{ds}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} g_{\mu\nu}.$$

The first integral of this equation of motion [1] is

$$e^{-2\int g_{\mu\nu}\frac{dx^\mu}{ds}dx^\nu} g_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = \text{constant}, \tag{1.10}$$

which can be easily verified by direct differentiation with regard to $s$: an equation which we see at once from its form is invariant under a transformation of coördinates since this does not alter the parameter of the path, namely, $s$.

The equation (1.10) leads to some interesting considerations concerning the types of periodic paths admitted by the equations of motion. Whereas the first integral of (1.3) gives only the condition that the $g$'s shall be functions of position in order to be periodic, *i.e.*, the particle return to the same position (same values of the $g$'s) with the same velocities $\left(\text{same values of } \frac{dq^i}{ds}\right)$, the first integral of (1.9) demands that in addition

$$e^{-2\int g_{\alpha\beta}\frac{dx^\alpha}{ds}dx^\beta}$$

[1] EISENHART, *Trans. Am. Math. Soc.*, **26**, 3, p. 378.

be periodic, *i.e.*, $$\int g_{\alpha\beta}\frac{dx^\alpha}{ds}\,dx^\beta = \pi n i \qquad \begin{cases} i = \sqrt{-1} \\ n = \text{integer} \end{cases} \tag{1.11}$$

the integral being taken over a complete cycle.

Before we discuss this condition further, we must know something of the conditions under which equations (1.3) and (1.9) admit of complete integration. Stackel has shown [2] that if the equations (1.6) and (1.10) can be written in the form

$$A_{\alpha\alpha}\left(\frac{\partial W}{\partial q^\alpha}\right)^2 = \text{constant}$$

and if these equations can be integrated by assuming that

$$W = Q^1(x^1) + Q^2(x^2) + \cdots\cdots Q^n(x^n),$$

then the motion can be completely described.

This method of solution by the separation of variables enables us when it is feasible to simplify the equation (1.11) since in this case, where $g_{ii}\left(\frac{dx^i}{ds}\right)$ – function of $x^i$ only, the integral (1.11) can be separated into $n$ distinct integrals, each of which must satisfy a relation of the form

$$\int g_{ii}\left(\frac{dx^i}{ds}\right)dx^i = \pi n_i i/n_i = \text{integer:} \tag{1.12}$$

these integrals are analogous in form to the Sommerfeld quantum conditions.

In order to explain this method more thoroughly, we shall consider the form of the equations (1.3) in the case of a particle with three degrees of freedom under the action of a force $F$ acting in the direction of the radius vector from the origin in the plane perpendicular to $x^3$ and of magnitude given by

$$F = \frac{m}{r^2}\left(\frac{dx^3}{ds}\right)^2 \tag{1.13}$$

$r$ being the distance from the origin along the radius victor lying in that plane.

Assuming that the first integral of the equations of motion can be written in the form for polar-cylindrical coördinates

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\theta}{ds}\right)^2 - e^f\left(\frac{dx^3}{ds}\right)^2 = \text{constant}$$

where the function $f$ must be so determined that the only force acting on the system is $F$.

[2] CHARLIER, "Die Mechanik des Himmels."

Knowing the values of the $g$'s, namely,

$$g_{11} = 1 \qquad g_{22} = r^2 \qquad g_{33} = -e^f,$$

the others being zero, and substituting these values into the equation defining the Γ's, we find the three equations of motion:

(i) $\frac{d^2r}{ds^2} - \frac{1}{2}\left(\frac{dr}{ds}\right)^2 - r\left(\frac{d\theta}{ds}\right)^2 + \frac{1}{2}e^f\frac{\partial f}{\partial r}\left(\frac{dx^3}{ds}\right)^2 = 0$

(ii) $\frac{d^2\theta}{ds^2} + \frac{2}{r}\left(\frac{dr}{ds}\right)\left(\frac{d\theta}{ds}\right) = 0$ or $r^2\left(\frac{d\theta}{ds}\right) = \text{constant}$

(iii) $\frac{d^2x^3}{ds^2} + \frac{\partial f}{\partial r}\left(\frac{dr}{ds}\right)\left(\frac{dx^3}{ds}\right) = 0$ or $e^f\frac{dx^3}{ds} = \text{constant}.$

From the definition of force, we see that if we act on the particle with the force whose components are $f_1$, $f_2$ and $f_3$, such that the particle is brought to rest so far as the motion in the $(r, \theta)$ plane is concerned, then the force $f$ must approach the limiting value

$$f_1 = F \qquad f_2 = f_3 = 0.$$

Replacing the zeros on the right-hand side of the equations (i), (ii) and (iii) by $f_1$, $f_2$ and $f_3$, we obtain in the limit when

$$\frac{d^2r}{ds^2} = \frac{dr}{ds} = \frac{d^2\theta}{ds^2} = \frac{d\theta}{ds} = 0$$

the condition

$$\frac{1}{2}\frac{\partial f}{\partial r}\left(\frac{dx^3}{ds}\right)^2 = F = \frac{m}{r^2}\left(\frac{dx^3}{ds}\right)^2,$$

or

$$e^f = \text{constant} - \frac{2m}{r} = c - \frac{2m}{r}, \tag{1.14}$$

which determines the function $f$ uniquely, except for an integration constant.

This gives for the first integral the form

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\theta}{ds}\right)^2 - \left(c - \frac{2m}{r}\right)\left(\frac{dx^3}{ds}\right)^2 = \text{constant},$$

or replacing $\left(\frac{dx^3}{ds}\right)$ by its value from (iii)

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\theta}{ds}\right)^2 - k\left(c - \frac{2m}{r}\right)^{-1} = \text{constant}$$

or $$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\theta}{ds}\right)^2 - (kc)\left[1 + \frac{2m}{rc} + \text{terms of higher order}\right] = \text{constant}.$$

Setting $mk = M$, the mass of the attracting particle, we arrive at the familiar energy equation with the difference that the independent parameter defining the path is $s$ instead of the usual parameter $t$, the time, but as we are not sure of the true significance of the parameter $t$, the time, which enters into the Newtonian mechanics, this distinction is of no particular importance.

From the energy integral, we see that the quantity $m$ is not identical with the mass of the attracting particle $M$, but is only proportional to it, the constant of proportionality being the constant of integration of the third equation of motion. This arises from the form of the expression defining the force acting on the moving body,

$$F = \frac{m}{r^2}\left(\frac{dx^3}{ds}\right)^2.$$

In the case where the Lagrangian function is composed of terms depending on the square of the velocities, the forces acting on the system must always be of this form. If we introduce into the above equation the value for $\left(\frac{dx^3}{ds}\right)$, namely,

$$\left(\frac{dx^3}{ds}\right) = \sqrt{k}/\gamma \qquad \gamma = c - \frac{2m}{r},$$

we find that the force $F$ is

$$F = \frac{mk}{r^2}\left(1 - \frac{4m}{r}\right) + \text{higher order terms}$$

or

$$= \frac{M}{r^2}\left(1 - \frac{4m}{r}\right) + \text{higher order terms}, \qquad (1.15)$$

so that the force obeys the Newtonian law only to a first approximation. In the study of the Theory of Relativity, it has been remarked that the Einstein connection for the advance of the perihelion could be accounted for by assuming a force function of this type. Since the only means of determining the nature of the constants is through comparison with experiment, the result that the quantity $m$ is not the mass of the particle need not disturb us, if we keep in mind that wherever the expression $\frac{m}{\gamma^2}\left(\frac{dl}{ds}\right)^2$ appears, we must regard this constant quantity as the true mass of the system.

## ELECTRODYNAMICS.

Maxwell's equations of the electromagnetic field state the relation holding between the electric force $(X, Y, Z)$, the magnetic force $(L, M, N)$, the density $\rho$ of electric charge and the density

of the electric current $(\sigma_x, \sigma_y, \sigma_z)$, where

$$\sigma_x = \rho \frac{dx}{dt}\,;\ \sigma_y = \rho \frac{dy}{dt}\,;\ \sigma_z = \rho \frac{dz}{dt}\,.$$

If we take the velocity of light as unity and write for convenience $X^1 = X$, $X^2 = Y$, $X^3 = Z$ and $X^4 = t$, the components of the electric and magnetic forces $F_{ij}$ are given by the table

$F_{ij} \rightarrow i$, $\downarrow$

| | | | |
|---|---|---|---|
| $0$ | $-N$ | $M$ | $-X$ |
| $N$ | $0$ | $-L$ | $-Y$ |
| $-M$ | $L$ | $0$ | $-Z$ |
| $X$ | $Y$ | $Z$ | $0$ |

If we introduce further the notation

$$\rho = \rho_0 \frac{dt}{ds}$$

and

$$J^i = \rho_0 \frac{dx}{ds}$$

then Maxwell's equations are in rectangular Carterian coördinates

$$\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x} = 0, \tag{2.1}$$

and

$$\frac{\partial F^i{}_\alpha}{\partial x^\alpha} = J^i, \tag{2.2}$$

where the $F^{ij}$'s are defined by means of the relation

$$g^{i\alpha} g^{j\beta} F_{\alpha\beta} = F^{ij},$$

the $g$'s in this case having the values

$$g^{11} = g^{22} = g^{33} = -1 \qquad g^{44} = 1 \qquad \text{and } g^{ij} = 0 \qquad i \neq j,$$

the $g$'s being transformed by a change in the coördinate system according to the relation given in the preceding section

$$g'^{ij} = g^{\alpha\beta} \left(\frac{\partial x^i}{\partial q^\alpha}\right) \left(\frac{\partial x^j}{\partial q^\beta}\right).$$

In order to complete the specification of electrodynamical phenomena, we must add to these equations the relation between the force acting on a charge in an electromagnetic field and the fundamental quantities defining the field. This force—the ponderomotive force—is given by the relation

$$h_i = F_{i\alpha} J^\alpha. \tag{2.3}$$

The values of the $g$'s used to raise or lower the suffixes are derived from the form of the energy equation in the limiting case of an electric particle emitting no radiation and under the action of no forces—the energy integral of the geodesic equation.

Since in this case this integral has the form

$$\left(\frac{dx^1}{ds}\right)^2+\left(\frac{dx^2}{ds}\right)^2+\left(\frac{dx^3}{ds}\right)^2-\left(\frac{dx^4}{ds}\right)^2=\text{constant},$$

the $g$'s have the above values.

Let us next consider the form of the equations of motion in the case of the motion of a charged particle possessing two degrees of freedom under the action of a charge at the origin of the spatial coördinate system, the reaction of its own field on itself being neglected for the present, this calculation being carried out in order to determine the form of the $g$'s used to raise or lower the indices.

Let us first see whether it is possible to write the equations of motion of the charged particle of mass $m$ and charge $e$ under the action of the charge $E$ at the origin of the spatial coördinate system in the form

$$m\left(\frac{d^2x^i}{ds^2}+\Gamma^i_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}\right)=0,$$

where the $\Gamma$'s must be so determined that the electric forces necessary to bring the particle to rest in the spatial coördinate system are equal in the limit to the electric forces of attraction between the two charges.

If we assume as before that the energy equation can be written in the form

$$\left(\frac{dr}{ds}\right)^2+r^2\left(\frac{d\theta}{ds}\right)^2-e^f\left(\frac{dt}{ds}\right)^2=\text{constant},$$

then the equations of motion are

(i) $$\frac{d^2r}{ds^2}+\frac{1}{2}\left(\frac{dr}{ds}\right)^2-r\left(\frac{d\theta}{ds}\right)^2+\frac{1}{2}e^f\frac{\partial f}{\partial r}\left(\frac{dt}{ds}\right)^2=0$$

(ii) $$\frac{d^2\theta}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds}=0 \qquad \text{or} \qquad r^2\frac{d\theta}{ds}=\text{constant}$$

(iii) $$\frac{d^2t}{ds^2}+\frac{\partial f}{\partial r}\frac{dr}{ds}\frac{dt}{ds}=0 \qquad \text{or} \qquad e^f\frac{dt}{ds}=\text{constant}=\kappa.$$

If we wish to make the path of the particle correspond to that

derived from the classical theory for a particle radiating no energy, then the limiting value of the forces necessary to bring the particle to rest in the $(r, \theta)$ plane must have the components

$$f_r = \frac{eE}{r^2} \qquad f_\theta = 0 \qquad f_t = 0,$$

so that we must have

$$\frac{1}{2}\frac{\partial e^f}{\partial r}\left(\frac{dt}{ds}\right)^2 = f_r = \frac{eE}{r^2};$$

but since

$$e^f \frac{dt}{ds} = \kappa,$$

$$\frac{\partial e^{-f}}{\partial r} = \frac{2eE}{r^2\kappa^2} \quad \text{or} \quad e^{-f} = \text{constant} + \frac{2eE}{r\kappa^2}$$

and the energy integral takes the form

$$m\left[\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\theta}{ds}\right)^2\right] - \left(c + \frac{2eE}{r\kappa^2}\right)^{-1}\left(\frac{dt}{ds}\right)^2 = \text{constant}.$$

This equation can also be written in the form

$$m\left[\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\theta}{ds}\right)^2\right] - \frac{2eE}{r} = \text{constant} + c\left(\frac{dt}{ds}\right)^2$$

to a first approximation, a form which is more useful for computation on account of its similarity to the classical energy equation.

Let us next consider the case where the particle, besides being under the influence of a central charge, is also reacted on by its own field which is specified by the $F$'s as solutions of Maxwell's equation.

The equations of motion of the charge are then

$$\frac{d^2q^i}{ds^2} + \Gamma^i_{\alpha\beta}\frac{dq^\alpha}{ds}\frac{dq^\beta}{ds} = F^i{}_\alpha \frac{dq^\alpha}{ds}, \tag{2.4}$$

where $F^i{}_j$ are the components of the electromagnetic field of the moving charge and where the $g$'s defined by means of the energy equation are used to raise or lower the indices, the $g$'s being chosen in such a way that the motion of the moving charge, neglecting the influence of its own field on itself, is a geodesic, *i.e.*, the above equation with $F^i{}_\alpha \frac{dx^\alpha}{ds} = 0$.

If we now make a transformation of coördinates

$$q^i = q^i(x^1, x^2 \cdots x^n),$$

then the equation (2.4) becomes

$$\frac{\partial q^i}{\partial x^\gamma}\left(\frac{d^2x^\gamma}{ds^2}+\dot{\Gamma}^\gamma_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}\right)=F^i{}_\alpha\frac{\partial q^\alpha}{\partial x^\delta}\frac{dx^\delta}{ds}$$

$$=F^{i\alpha}g_{\alpha\beta}\frac{\partial q^\beta}{\partial x^\delta}\frac{dx^\delta}{ds}.$$

But if this equation is invariant for a transformation of coördinates,

$$\frac{d^2x^i}{ds^2}+\dot{\Gamma}^i_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}=\dot{F}^{i\alpha}g'_{\alpha\beta}\frac{dx^\beta}{ds},$$

so that

$$\frac{\partial q^i}{\partial x^\nu}\left(\dot{F}^{\gamma\alpha}g_{\mu\nu}\frac{\partial q^\mu}{\partial x^\alpha}\frac{\partial q^\nu}{\partial x^\beta}\right)\frac{dx^\beta}{ds}$$

$$=F^{i\alpha}g_{\alpha\beta}\frac{\partial q^\beta}{\partial x^\delta}\frac{dx^\delta}{ds},$$

or

$$\dot{F}^{ij}=F^{\alpha\beta}\frac{\partial x^i}{\partial q^\alpha}\frac{\partial x^j}{\partial q^\beta}, \tag{2.5}$$

which shows that $F$ is transformed according to the same law as the $g$'s, *i.e.*, $F$ is a tensor.

We shall now consider the form of Maxwell's equations in the spherical cylindrical coördinate system we have been using

$$x^1=r \qquad x^2=\theta \qquad x^3=\phi \qquad x^4=t$$

and where $ds$ is the measure of the element of distance.

Since the components of the electromagnetic field $F$ are transformed according to the tensor law, we should expect that these equations could be expressed in a simpler form by the use of covariant differentiation just as we have seen that the equations of motion of a gravitational particle could be expressed in the simple form

$$\left(g_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}\right)_\gamma\frac{dx^\gamma}{ds}=0 \tag{2.6}$$

The appropriate form of Maxwell's equations [3] in any type of coördinate system is

$$(F^{i\alpha})_\alpha=G\cdot J^i$$

and

$$F_{ijk}+F_{jki}+F_{kij}=0, \tag{2.7}$$

where the $g$'s are used to raise or lower the indices.

Since the components of $F$'s are skewsymmetrical, the second of these equations can be written in the more familiar form

[3] J. A. SCHOUTEN, "Der Ricci Kalkul."

$$\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0,$$

for the terms containing the $\Gamma$'s mutually cancel.

If we take the covariant derivative of the first equation with respect to $i$ and sum, we obtain on account of the skewsymmetry of $F$, the equation

$$(GJ^\alpha)_\alpha = 0 \qquad (2.8)$$

which represents the equation of continuity in this coördinate system.

It will be noticed that we have written the first set of Maxwell's equations with a multiplying factor $G$ on the right-hand side, instead of in the form

$$(F^{i\alpha})_\alpha = J^i,$$

which is the generalized form of Maxwell's equations in the Riemannian coördinate system, where the independent parameter is assumed to have a uniquely determined value at every point in the space. In the work so far we have not considered the physical meaning of this parameter—if such exists—the reason being that we have tacitly assumed that the results we have obtained, namely, the energy function of the moving particle, are independent of the value of the parameter $s$. This assumption is equivalent to one of the basic hypotheses of Weyl's geometry, that

$$g_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}$$

is independent of the form of $ds$, *i.e.*, if we arbitrarily change the value of the unit of measure of the standard at each point in such a way that in the new measuring system, marked by accent quantities,

$$ds' = \lambda ds.$$

Since the components of $dx$ are independent of the parameter,

$$dx'^i = dx^i,$$

we must have, in order to preserve the invariance of the energy function,

$$g'_{ij} = \lambda^2 g_{ij}$$

which gives the mode of transformation of the $g$'s with regard to a change in the unit of measure of the parameter $S$.

We shall now consider the necessary form of the equations

of motion in order that they be invariant for this type of transformation—gauge transformation.

Transforming the $\Gamma$'s by a change of gauge, we have

$$\acute{\Gamma}_{ij,k} = \frac{1}{2}\left(\frac{\partial(\lambda^2 g_{ik})}{\partial x^j} + \frac{\partial(\lambda^2 g_{jk})}{\partial x^i} - \frac{\partial(\lambda^2 g_{ij})}{\partial x^k}\right)$$
$$= \lambda^2 \Gamma_{ij,k} + \lambda^2[g_{ik}\phi_j + g_{jk}\phi_i - g_{ij}\phi_k].$$

Multiplying through by $\acute{g}^{\kappa k} = \lambda^{-2} g^{\kappa k}$, we obtain

$$\acute{\Gamma}^{\kappa}_{ij} = \Gamma^{\kappa}_{ij} + \delta^{\kappa}_i \phi_j + \delta^{\kappa}_j \phi_i - g_{ij}\phi_\kappa$$

where we have set

$$\frac{\partial \log \lambda}{\partial x^i} = \phi_i .$$

If we then set

$$^*\Gamma^{\kappa}_{ij} = \Gamma^{\kappa}_{ij} - \delta^{\kappa}_i \phi_i - \delta^{\kappa}_j \phi_i + g_{ij}\phi^{\kappa}, \tag{2.9}$$

the generalized "three index symbol" $^*\Gamma^{\kappa}_{ij}$ is invariant for a transformation of the independent parameter, and the equations of motion in their invariant form become

$$\frac{d^2 x^i}{ds^2} + {}^*\Gamma^{i}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \tag{2.10}$$

whose first integral, as we have already seen, has the form

$$e^{-2\int \phi_\mu dx^\mu} g_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = \text{constant}. \tag{2.11}$$

Since the $F_{ij}$'s, the components of the electric and magnetic forces, are pure numbers, they are independent of a change of gauge. On this account the second set of Maxwell's equations,

$$\frac{\partial F_{ij}}{\partial x^\kappa} + \frac{\partial F_{j\kappa}}{\partial x^i} + \frac{\partial F_{\kappa i}}{\partial x^j} = 0,$$

are invariant, as the first set must also be if they have any physical significance. Written in the form

$$\left(g^{i\mu} g^{\alpha\nu} F_{\mu\nu}\right)_\alpha = J^i,$$

they are no longer invariant, but if we multiply the right-hand side by a factor $G$, we can choose this function in such a manner that the first of equations (2.7) is also invariant.

Setting $\acute{g}^{ij} = \lambda^{-2} g^{ij}$, we obtain, since

$$\acute{F}^{ij} = g^{i\alpha} g^{j\beta} F_{\alpha\beta} = \lambda^{-4} F^{ij}$$

and
$$(F^{i\alpha})_{\alpha} = \frac{\partial F^{i\alpha}}{\partial x^{\alpha}} + \Gamma^{i}_{\alpha\beta} F^{\alpha\beta} + \Gamma^{\alpha}_{\beta\alpha} F^{i\beta},$$

$$(\dot{F}^{i\alpha})_{\alpha} = (F^{i\alpha})_{\alpha}\lambda^{-4} + \lambda^{-4}[4F^{i\alpha}\phi_{\alpha} - 4F^{i\alpha}\phi_{\alpha} - F^{\alpha\beta} g_{\alpha\beta}\phi^{i}]$$

$$= (F^{i\alpha})_{\alpha}\lambda^{-4} - F^{\alpha\beta} g_{\alpha\beta}\phi^{i}\lambda^{-4}.$$

If we now limit the discussion to the case where

$$g_{ij} = 0 \qquad i \neq j,$$

since $F^{ij}$ is skewsymmetric, $F^{\alpha\beta} g_{\alpha\beta}$ is identically zero, so that

$$(\dot{F}^{i\alpha})_{\alpha} = (F^{i\alpha})_{\alpha}\lambda^{-4}. \tag{2.12}$$

From this equation we see that the right-hand side of the first set of Maxwell's equations, $J^{i} = \rho \frac{dx^{i}}{ds}$, can be replaced in the more general case by

$$J^{i} = G\rho \frac{dx^{i}}{ds}$$

where $G\rho$ is a function varying as $\lambda^{-3}$.

With this understanding the equations

$$(F^{i\alpha})_{\alpha} = G\rho \frac{dx^{i}}{ds} \tag{2.13}$$

are invariant for a gauge transformation.

### RELATION OF MAXWELL'S EQUATIONS TO THE EQUATION OF MOTION OF A CHARGED BODY.

As we have seen, the first integral of the geodesic equations which have the form of an invariant for a change in gauge can be written in the form

$$e^{\int_{2} \phi_{\alpha} dx^{\alpha}} g_{\mu\nu} \frac{dx^{\mu}}{ds} \frac{dx^{\nu}}{ds} = \text{constant},$$

or if the $g$'s satisfy the condition

$$g_{ij} = 0 \qquad i \neq j,$$

$$\frac{e}{g_{\mu\mu}}^{\int_{2} \phi_{\alpha} dx^{\alpha}} \left( g_{\mu\mu} \frac{dx^{\mu}}{ds} \right)^{2} = \text{constant}.$$

The complete solution of this differential equation of motion can be found by the method of separation of variables (Stackel's solution), provided this equation can be written in the form

$$A_{\alpha\alpha}\left(\frac{\partial W}{\partial x^{\alpha}}\right)^{2} = \text{constant},$$

where the $A$'s are functions of the $x$'s and

$$W = Q_1(x^1) + Q_2(x^2) + \cdots + Q_n(x^n),$$

$Q_1(x^i)$ being a function of $x^i$ only.

If, therefore, $g_{ii}\frac{dx^i}{ds}$ can be written in the form

$$g_{ii}\frac{dx^i}{ds} = \frac{\partial W}{\partial x^i}$$

the complete solution of the problem can be found.

Now in the coördinate system with arbitrary gauge, there exists besides the fundamental form $g_{ij}$, the gauge factor $\lambda$. Various identifications of this factor have been proposed, but we shall assume here that we can regard the gauge factor, $\log \lambda$, as identical with the integration function $W$, an equality which removes the arbitrariness in the measuring system and gives a method of uniquely determining the measure of distance at each point of the manifold.

If we set, therefore,

$$\log \lambda = W = Q_1 + Q_2 + \cdots Q_n,$$

then the first integral has the form

$$\frac{e^{2\int \frac{\partial W}{\partial x^{\alpha}} dx^{\alpha}}}{g_{\mu\mu}} \cdot \left(\frac{\partial W}{\partial x^{\mu}}\right)^{2} = \text{constant},$$

where $\frac{\partial W}{\partial x^i}$ represents the covariant component of the momentum in the $x^{i\,\text{th}}$ direction:

$$\frac{\partial W}{\partial x^i} = g_{ii}\left(\frac{dx^i}{ds}\right) = p_i,$$

so that the integral can be written in the form

$$e^{2\int p_{\alpha} dx^{\alpha}} \cdot p_{\beta}p^{\beta} = \text{constant}. \tag{3.1}$$

With this interpretation of the function $\lambda$,

$$^{*}\Gamma^{\kappa}_{ij} = \Gamma^{\kappa}_{ij} - \delta^{\kappa}_{i}p_j - \delta^{\kappa}_{j}p_i + g_{ij}p^{\kappa} \tag{3.2}$$

and

$$p_{\alpha}p^{\alpha} = \frac{g^{\alpha\alpha}}{\lambda^2}\left(\frac{\partial \log \lambda}{\partial x^{\alpha}}\right)^{2}$$

a form which is no longer invariant for a change in gauge.

As we shall see, the equations of motion in this case

$$\frac{d}{ds}(p^i) + {}^*\Gamma^i_{\alpha\beta}\, p^\alpha p^\beta = 0, \tag{3.3}$$

have an important relationship with regard to the electrodynamical equations of motion of a charge. In the mechanical situation we have been considering, we have no means of evaluating the gauge factor $\lambda$, except through the equations of motion of the particle. If, however, we can represent some physical process by means of the equations of motion in this form, then we are in a position to determine completely the character of the manifold. The difference in the invariant properties of

$$g_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}$$

and the weights of the other quantities is due to the choice of the measure of distance along these paths being no longer the same as the geodesic measure which preserves the invariance of the above quantity, for the equations (3.1 and 3.2) have been derived on the supposition that among all the infinite number of possible values for the gauge factor $\lambda$, a definite set has been chosen in such a way that

$$p_i = \frac{\partial \log \lambda}{\partial x^i} = \phi_i .$$

If we were to make a change in the value of the gauge vector $\phi_i$, then the above condition would no longer hold true, so that we cannot expect these relations to have an invariant form with respect to a change of gauge. By the weight of a quantity, we mean the exponent of the power of $\lambda$ by which this quantity is multiplied when we alter the gauge, *e.g.*, $g_{ij}$ is of weight two: $\frac{dx^i}{ds}$ of weight minus one.

Before we proceed further with the discussion of this equation, we shall return to a consideration of the type of solution of Maxwell's equations written in the form (2.13), in order to find the form of the path of a radiating charge revolving about a central charge.

The electromagnetic equations have generally been considered as determining the nature of the electromagnetic field when the motion of the charge is known. If, however, we were in a position to represent the electromagnetic forces on the surface and

within the charge as given functions of the velocities and position of that charge, then Maxwell's equations could be regarded as the equations of motion of the moving charge. In this paper we shall use this second method in order to attack the problem of the radiating electron.

The electromagnetic equations for any coördinate system whose manifold is determined by a set of $g$'s and a gauge factor $\lambda$, can be written in the form

$$\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0 \tag{3.4}$$

and

$$(F^{i\alpha})_\alpha = J^i$$

where

$$J^i = G\rho \frac{dx^i}{ds}$$

the factor $G$ being unity.

We shall now seek to define the forces acting on the surface and within the charge in such a manner that the first set of Maxwell's equations is identically satisfied while the second set becomes the equations of motion of the moving charge in the form required by the quantum mechanics. Since the only case where we can completely determine the solution of the equations of motion is when the variables are separable, we shall suppose that the latter condition is fulfilled. This necessitates, since the equations of motion must be invariant for a change of gauge, that the equations of the moving charge have the form

$$\frac{dp^i}{ds} + {}^*\Gamma^i_{\alpha\beta} p^\alpha p^\beta = 0, \tag{3.5}$$

where

$${}^*\Gamma^\kappa_{ij} = \Gamma^\kappa_{ij} - \delta^\kappa_i \phi_j - \delta^\kappa_j \phi_i + g_{ij}\phi^\kappa,$$

$p^i = i^{\text{th}}$ component of the momentum vector

and

$\phi_i = i^{\text{th}}$ component of the gauge vector,

the $\Gamma$'s being determined so that the equations of motion of the moving charge, when the influence of its own field on the charge is neglected, have the form

$$\frac{dp^i}{ds} + \Gamma^i_{\alpha\beta} p^\alpha p^\beta = 0,$$

a form whose first integral is

$$g_{\alpha\beta} p^\alpha p^\beta = \text{constant} = H \tag{3.6}$$

and which has already been discussed. The function $H$, which appears as the constant of integration, is defined as the energy of the system: A definition which agrees with the Bohr-Sommerfeld postulation of this important entity, and which with the $g$'s and $p$'s as determined will suffice to account completely for the properties of the motion of a moving particle—no radiation considered—under the action of a given external field. However, before we attempt to discuss this equation (3·5), we shall first seek to define the nature of the forces acting on the surface and within the charge in such a fashion that the conditions that (i) the first set of Maxwell's equations is satisfied identically, and (ii) the second set becomes identical with the equation of motion (3·5), are justified.

The separability of the equations of motion requires that

$$p_i = g_{i\alpha}\frac{dx^i}{ds} = Q_i(x^i),$$

*i.e.*, the covariant components of the momentum vector are functions of the corresponding coördinates only.

The first set of Maxwell's equations is then satisfied if we set

$$F_{ij} = Q_iQ_j \cdot I_{ij}, \tag{3·7}$$

where the components of the momentum vector $Q_i$ are supposed to be divided by the unit of measure in order to retain the $F_{ij}$'s as mere numbers with respect to a change in gauge and where the constant quantities $I_{ij}$ have the skewsymmetric property that

$$I_{ij} = -I_{ji} \text{ and } I_{ii} = 0. \tag{3·8}$$

The $I_{ij}$'s have the appropriate signs so that the scheme for the $F$'s (page 782) remains the same, and they have the appropriate dimension, so that the units involved are the same for the two sides of the equation.

The second set becomes $(g^{i\alpha}Q_\alpha g^{\mu\beta}Q_\beta I_{\alpha\beta})_\mu = G\rho Q^i$,

where $$Q^i = g^{i\alpha}Q_\alpha,$$

or $$(g^{i\alpha}Q_\alpha g^{\mu\beta}Q_\beta)_\mu \cdot I_{\alpha\beta} = G\rho Q^i,$$

which represents the equations of motion of the moving charge under the action of its own field and the external field defined by the $g$'s. If we take the case where

$$g_{ij} = 0 \quad i \neq j,$$

then the equation becomes

$$(Q^i Q^\mu)_\mu I_{i\mu} = G\rho Q^i$$

or

$$(Q^i Q^\mu{}_\mu + Q^\mu Q^i{}_\mu) I_{i\mu} = G\rho Q^i,$$

subject to the two conditions: (i) That the relation $J^\alpha_\alpha = 0$ is satisfied, and (ii) that the differential equation is independent of the gauge system, since it represents the second of Maxwell's equations.

Since the invariant $G\rho$ is subject to no restriction regarding the distribution of charge density, we can assume that it satisfies the relationship

$$(G\rho)_\mu Q^\mu = 0,$$

provided it is compatible with the two conditions, (i) and (ii).

Since the first requires $J^\alpha_\alpha = (G\rho Q^\alpha)_\alpha = 0$, we have $Q^\alpha_\alpha = 0$, so that the equation of motion becomes

$$(Q^i{}_\alpha Q^\alpha) I_{i\alpha} = G\rho Q$$

Before we proceed further with this discussion, we shall have to consider more fully the meaning of the standard of length with which we measure the different quantities involved. In order to carry out our measurements at each point in space, we must have a standard associated with each given point. Suppose we consider the standard of length $l$ at the point $A$ $(x, y, z)$ in the space, and we wish to compare this with the standard at an infinitesimally distant neighboring point $B$ $(x + dx, y + dy, z + dz)$, taking for our illustration a three-dimensional manifold. Since in order to compare the two standards, we must displace the standard of length, $l$, from $A$ to $B$ by exerting a system of forces on it, the standard $l$ may have changed in length during this operation. In order then to compare these two quantities, we shall have to know the change in the standard for the infinitesimal displacement which we shall postulate to be of the form

$$d(\log l) = \kappa_\alpha dx^\alpha, \tag{3.9}$$

where $\kappa_i$ = point function.

This definition enables us, when we know the $\kappa_i$'s, to compare the standards of measurement through the whole space.

If we now consider the standard at $A$, lying along the $x'$ axis, and rotate it, for example, into coincidence with the $x^2$ axis, we must also exert a system of physical forces on the measure which may appreciably alter its length. In order then to compare the

standard in different orientation at the same point, we must know the law governing the change of length with orientation. Specifying any given orientation by the angles $\theta^1$ and $\theta^2$, we shall assume the change of length, when we alter the direction of the standard by a small amount, to be given by

$$d(\log \lambda) = \lambda_\alpha d\theta^\alpha. \tag{3.10}$$

In order then to compare two lengths at two different points in different directions, varying infinitesimally from each other, we first rotate one of the standards until the two are parallel and then displace one of them parallel to itself and to the other until they coincide; since the change in length is known for these two operations, the comparison can be uniquely carried out.

If we consider that the phenomena of electromagnetism are manifested in a space with this twofold indeterminateness in the calibration of the units of length, then we must, in order completely to determine the manifold, evaluate uniquely the $\lambda$'s and $\kappa$'s at each point of the space.

Suppose now we take a standard of measure at each point in space in such a way that the length depends on the orientation

$$d(\log l) = \kappa_\alpha d\theta^\alpha,$$

the $\kappa$'s being point functions. Then the values of the velocities, being vector functions, will depend on the particular rotational measuring system used.

If we now make an arbitrary transformation in this measuring system by changing the values of the $\kappa$'s, the value of the velocity in any direction $Q_i$ in the new system will be related to the value $Q_i$ in the former system by the equation of the form

$$\dot{Q}^i = \triangle^i Q^i, \tag{3.11}$$

the transformation being chosen of such a nature that the $\triangle$'s are constants. Since the quantity

$$g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds}$$

must be invariant for this transformation, the $g$'s must change in the following manner,

$$\dot{g}_{\alpha\beta} = g_{\alpha\beta} / \triangle^\alpha \triangle^\beta,$$

in order to preserve the invariance of this expression.

If we designate the quantities in the transformed system by primes, then

$$\acute{F}^{ij} = \acute{Q}^i \acute{Q}^j I_{ij} = Q^i Q^j (\triangle^i \triangle^j I_{ij})$$

and

$$\begin{aligned} \acute{F}_{ij} &= \acute{g}_{ii}\, \acute{g}_{jj}\, \acute{F}^{ij} \\ &= g_{ii}\, g_{jj}\, Q^i Q^j I_{ij}, \end{aligned} \tag{3.12}$$

since the $g$'s are divided by the measuring unit in this expression. The $F_{ij}$'s are, therefore, invariant for this type of transformation. With regard to the new calibration, we can determine the $F^{ij}$'s in such a manner that

$$\acute{F}^{ij} = cQ^i Q^j \qquad \acute{F}^{ii} = 0, \tag{3.13}$$

where $c$ is a constant which we will take as unity.

The determination of the $\triangle$'s in this manner is the same as the problem of determining the basal units of a four-dimensional quaternion system with non-vanishing determinant, since it is only the product of the factors whose value is preassigned.

For a three-dimensional manifold, we can easily find a set of $\triangle$'s which will enable us to transform the equations

$$(Q^i{}_\alpha Q^\alpha)\, I_{i\alpha} = G\rho Q^i$$

into the form

$$(\bar{Q}^i{}_\alpha \bar{Q}^\alpha) = \bar{G}^i \bar{Q}^i$$

If we give the $\triangle$'s the following values,

$$\triangle^1 = \begin{vmatrix} \sqrt{-1} & 0 \\ 0 & -\sqrt{-1} \end{vmatrix} \qquad \triangle^2 = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$$

and

$$\triangle^3 = \begin{vmatrix} 0 & \sqrt{-1} \\ \sqrt{-1} & 0 \end{vmatrix},$$

then

$$\begin{aligned} &\triangle^1 \triangle^2 = \triangle^3 \qquad && \triangle^2 \triangle^1 = -\triangle^3 \\ &\triangle^1 \triangle^3 = -\triangle^2 \qquad && \triangle^3 \triangle^1 = \triangle^2 \\ &\triangle^2 \triangle^3 = \triangle^1 \qquad && \triangle^3 \triangle^2 = -\triangle^1 \end{aligned}$$

and

$$(\triangle^1)^2 = (\triangle^2)^2 = (\triangle^3)^2 = -1 \qquad 1 = \text{unit matrix}$$

Making the transformation

$$Q^i = \bar{Q}^i \triangle^i$$

we obtain, if we chose initially the constant skewsymmetric quantities $I_{ij}$, in the following manner,

$$I_{ij} = \triangle^\kappa$$

and
$$I_{ji} = -\Delta^{\kappa} \qquad i \neq j \neq \kappa$$
$$(\bar{Q}^i{}_{\alpha}\bar{Q}^{\alpha}) = -G\rho\Delta^i\bar{Q}^i$$
$$= \bar{G}^i\bar{Q}^i$$

This system of basal quaternion units,[4] which we have used as the factors in this contact transformation, can be generalized to the four-dimensional case so that this type of transformation can be carried out legitimately.

If we now limit our problem in the rotational system, determined in the above manner by taking the particular solution of the equation

$$Q^{\alpha}{}_{\alpha} = 0,$$

namely,
$$Q^i{}_i = 0, \qquad i \times 1 \cdots 4 \tag{3.14}$$

the equation of motion has the form

$$Q^{\mu}Q^i{}_{\mu} = G\rho Q^i\Delta^i,$$

or, writing
$$G^i = G\rho\Delta^i,$$
$$Q^{\mu}Q^i{}_{\mu} = G^iQ^i = G^iQ^i(Q_{\alpha}Q^{\alpha}). \tag{3.15}$$

When we write the equations of motion of the moving charge in the form

$$Q^{\mu}Q^i{}_{\mu} = G^iQ^i$$

we see that there does not appear in these equations any quantity corresponding to the mass of the moving charge. In order then to make these equations assume a form analogous to those used in quantum mechanics, we shall define the scalar function

$$g_{\alpha\beta}\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = \mu$$

as the mass density of the system, a definition which is the same as that used in the general relativity discussion of the properties of matter.[5]

Multiplying both sides of the equation (3.15) by this expression, we obtain the final equation

$$\mu Q^{\mu}Q^i{}_{\mu} = G^iQ^i(Q_{\alpha}Q^{\alpha}),$$

or
$$Q^{\mu}Q^i{}_{\mu} = \bar{G}^iQ^i(Q_{\alpha}Q^{\alpha}),$$

[4] DICKSON, "Algebras and Their Arithmetics," p. 19.
[5] EDDINGTON, p. 121.

where $$\bar{G}^i = G^i/\mu,$$

a generalization of the equations of the geodesics of a Riemannian space, where the vector $\phi_i$ corresponds to the momentum vector of the motion subject to the restriction

$$\left(\frac{dx^i}{ds}\right)_i = 0.$$

The condition (i) reduces, in this case, to the following restriction,

$$(G\rho)_\mu Q^\mu = 0$$

or $$G\rho_\mu Q^\mu = 0.$$

Hence, $Q^\mu \rho_\mu = 0$, an equation which gives the distribution of charge density in the body in order that the condition (i) may be satisfied. Since we are ignorant of the character of this distribution, we can take the fulfilment of this condition as granted. The only arbitrary condition which has been placed on the solution is that the equation

$$Q_i^i = 0$$

shall be true for the type of motion considered.

The first integral of the equation of motion (3.15) is

$$g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)^2 e^{-2\int \bar{G}^\beta Q^\beta Q_\beta ds} = \text{constant}, \tag{3.16}$$

where the integral is taken along the path of the moving charge. Since we have defined the quantity,

$$H = g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)^2$$

as the energy corresponding to the motion, it follows that

$$H = \text{constant } e^{-2\int \bar{G}^\alpha Q^\alpha Q_\alpha ds}$$

which gives the energy at any point on the path.

If the paths are periodic, the integral must satisfy the condition

$$\int \bar{G}^\alpha g_{\alpha\alpha} \frac{dx^\alpha}{ds} dx^\alpha = \pi n i, \qquad \begin{cases} i = \sqrt{-1} \\ n = \text{integer} \end{cases} \tag{3.17}$$

the integral being taken over a complete cycle. When the variables are separable, this condition of periodicity reduces to a set

of equations analogous to Sommerfeld quantum conditions with the addition, however, of the quantum integral with respect to the time coördinate. This extra quantum condition has been used by the writer to give an account of the "Multiplet Structure of Lines"[6] and the relation of the "Multiplet and Zeeman Structures,"[7] giving formulæ for these phenomena which agree well with experiment. The fourth quantum condition has also been used by P. A. M. Dirac[8] to account on the wave theory for the Compton effect.

Since in the case where the motion is periodic the energy function has always the same value at any given point after a complete revolution, the motion may be classified as non-radiating.

From the above results, we are thus able to formulate the following theorem:

The path of a non-radiating electron obeying Maxwell's equations satisfies the equations of motion of a generalized Riemannian geodesic with the momentum taken as the gauge vector in a four-dimensional space subject to the two sets of conditions

$$\int \bar{G}^i p_i dq^i = n_i h,$$

where $n_i$ is an integer and $h$ a universal constant, $p_i$ being the momentum corresponding to the coördinate $q^i$, and

$$\left(\frac{dq^i}{ds}\right)_i = 0 \qquad i = 1 \cdots 4,$$

and the equations of motion being of such a form that a coördinate system can be chosen in which the variables are separable.

The quantity on the right-hand side of equation (3.15) represents the ponderomotive force $P^i$ of the field due to the electron on itself,

$$P^i = \bar{G}^i Q^j Q_\alpha Q_\alpha = \bar{P}^i F^i{}_\alpha Q^\alpha, \tag{3.18}$$

where the factor $\bar{P}^i$ is function of position. Although this quantity does not agree with Lorentz's definition of the ponderomotive force

$$P^i = F^i{}_\alpha Q^\alpha$$

[6] *Phil. Mag.*, June, 1925.
[7] JOUR. FRANK. INST., Jan., 1926.
[8] *Proc. Roy. Soc.*, June, 1926.

given in the preceding section (2.3), yet as this definition of this force is empirical, it is not surprising that the above derivation should lead to a different expression for this quantity. The results obtained by this method may be regarded as proving the existence of a four-dimensional manifold for which the reaction of the electromagnetic field of the electron, calculated by means of Maxwell's equations, due to itself is proportional to the velocity of the particle, the path of the particle neglecting this reaction being geodesic in the space.

### APPLICATION TO RADIATION.

In moving from one of the non-radiating paths to another, which we shall designate as quantum orbits, because the various paths correspond to definite values of the integers $n_i$ entering into the conditions of periodicity,

$$\int p_i dq^i = n_i h ;$$

the former with energy $H'$ and component of momentum, $p'_i$, the latter with energy $H''$ and component of momentum $p''_i$. The electron will emit radiation of energy $H' - H''$ and momentum $p'_i - p''_i$, the direction of the energy wave being determined by the momentum vector in order to preserve the law of conservation of energy and momentum for the total system, composed, on the one part, of the electron and its field, and, on the other, of the radiant energy.

Since we know now the equations of motion of the charge, we could, having solved the equations of motion, determine the nature of the electromagnetic field through the application of Maxwell's equations, the amount of energy radiated being given by Poynting's theorem. As the generalization of Poynting's theorem to our particular case must be such that the two laws of conservation are preserved, we may dispense with the actual solution by this method for the present object of our discussion.

The electromagnetic potentials $\kappa_i$, found as solutions of Maxwell's equations for the wave of radiant energy, will be of the form

$$\kappa_i = A_i e^{l_1 x^1 + l_2 x^2 + l_3 x^3 - l_4 x^4},$$

where $l_1$, $l_2$, $l_3$, and $l_4$, the direction cosines of the wave direction in the four-dimensional space, will be proportional to the components of the momentum vector of the radiation wave.

$$l_i \propto \text{momentum of wave in } x^i \text{ direction.} \tag{4.1}$$

Consider the surface defined by

$$\phi_1 = l_1x^1 + l_2x^2 + l_3x^3 - l_4x^4 = \text{constant and } t = x^4 = \text{constant},$$

which is the spatial wave front. If the wave is far enough from the source, then in free space the rays are everywhere normal to the hypersurface $\phi =$ constant.

Let the parametric form of the ray equation be given to the form

$$x^i = x^i(\alpha)$$

$\alpha$ being the parameter; then the partial derivatives

$$\frac{\partial \phi}{\partial x^i} = g_{ia}\frac{dx^\alpha}{ds} = \frac{dx^i}{da}$$

will represent the direction cosines of the ray, provided the $g$'s are Euclidean at that point.

If the disturbance is to be interpreted as wave motion, the phase of the wave must be capable of being expressed in the form

$$\phi = \nu\left(\frac{dx^1}{V}\cos\alpha + \frac{dx^2}{V}\cos\beta + \frac{dx^3}{V}\cos\gamma + dt + \delta\right),$$

where $\nu =$ the frequency of vibration

$V =$ velocity of propagation

$\cos\alpha$, $\cos\beta$ and $\cos\gamma =$ the direction cosines in the natural three-dimensional system used by the observer and $\delta =$ phase angle. On expanding $\phi$ in the terms of the coördinates

$$\phi = \phi_0 + \frac{\partial \phi}{\partial x^\alpha}dx^\alpha$$

and comparing, we get for the components of the direction cosines of the normal to the wave front,

$$\frac{\partial \phi}{\partial x^i} = \left(\frac{\nu\cos\alpha}{V}, \frac{\nu\cos\beta}{V}, \frac{\nu\cos\gamma}{V}, \nu\right), \tag{4.2}$$

so that the wave equation can be written in the form

$$\kappa_i = A_i e^{\frac{2\pi i\nu}{V}(\cos\alpha\cdot x + \cos\beta\cdot y + \cos\gamma\cdot z - Vt)}. \tag{4.3}$$

if the quantities $\frac{\partial \phi}{\partial x^i}$ are constant.

Similar attempts to identify the frequency of radiation as the time component of a vector whose properties are analogous to

the energy momentum tensor have been made by C. Eckhart [9] and DeBroglie.[10]

From this consideration, we see that in order to have interference phenomena, the direction of the two waves in the four-dimensional space must be identical, the nature of the interference, partial or total, depending on the relative phase of the two waves. If we write (4.3) in the familiar form

$$\kappa_i = A_i e^{\frac{2\pi i \nu}{V}(\cos\alpha x + \cos\beta y + \cos\gamma z - Vt)},$$

we are led to the following theorem:

The frequency $\nu$ of an electromagnetic wave is equal to the direction cosine between the direction of the wave normal and the time axis.

When the energy function is of the form

$$H = g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)^2,$$

the components of momentum in the $x^i$ direction are

$$Mi = \frac{2g_{ii}\left(\frac{dx^i}{ds}\right)^2}{\left(\frac{dx^i}{ds}\right)} = 2g_{ii}\left(\frac{dx^i}{ds}\right),$$

or $Mi$ is equal to the component of the energy due to the motion in the $x^i$ direction, namely, $g_{ii}\left(\frac{dx^i}{ds}\right)^2$, divided by one-half the velocity in that direction $\frac{1}{2}\left(\frac{dx^i}{ds}\right)$.

Now the electromagnetic momentum, whose components are defined through Poynting's vector for the three spatial directions as the flux of electromagnetic energy in those directions—for an Euclidean coördinate system at the point under consideration—divided by the square of the velocity of light, a definition which we shall generalize to include the momentum in the direction of the time axis as well. The law of conservation of electromagnetic momentum in free space is, therefore, equivalent to the conception of conservation of energy flux in the given direction. The process of radiation being a type of transformation of electro-

[9] *Phy. Rev.*, Dec., 1924.
[10] *Jour. de Phys.*, Jan., 1925.

magnetic energy into a different form, the law of conservation of energy flux will also hold in that phenomenon.

The direction of the normal to the wave front being the same as the direction of the propagation of energy flux, or electromagnetic momentum, the direction cosines of the wave normal can be taken as proportional to the components of the momentum vector along the various axes,

$$kM_i = \left( \frac{\nu \cos \alpha}{V}, \frac{\nu \cos \beta}{V}, \frac{\nu \cos \gamma}{V}, \nu \right)$$

so that

$$\nu = kM_4. \tag{4·4}$$

Now according to the law of conservation of electromagnetic momentum, or energy flux, we have

$$\nu = \frac{k}{c^2} \text{ (flow of electromagnetic energy in time direction)}$$

or $$\nu = \frac{k}{c^2} \text{ (flow of radiated energy in time direction)}$$

$$= \frac{k}{c}\left[ g_{44} \left( \frac{dx^4}{ds} \right)^2 \right]^{\prime\prime}$$

$$= \frac{k}{c}\left[ \frac{p_4^{\prime 2}}{g_{44}^{\prime}} - \frac{p_4^{\prime\prime 2}}{g_{44}^{\prime\prime}} \right], \tag{4·5}$$

the only possible direction for the energy flux being in the direction of increasing time as regards this coördinate, where the above $p$'s and $g$'s refer to the motion of the electron in the two quantized orbits, respectively.

The energy $H$ of a non-radiating electron moving in a plane with regard to the three-dimensional spatial space about a central body at the origin of the spatial manifold of charge $E$ is

$$-H = \left(1 - \frac{2eE}{r}\right)^{-1} \left(\frac{dr}{ds}\right)^2 + r^2 \left(\frac{d\theta}{ds}\right)^2 - \left(1 - \frac{2eE}{r}\right) \left(\frac{dt}{ds}\right)^2$$

$$= g_{\alpha\alpha} \left( \frac{dx^\alpha}{ds} \right)^2,$$

using a cylindrical polar coördinate system.

The equations of motion of the electron

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = G^i g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds}, \frac{dx^i}{ds}$$

take the following forms in the special case of a circular orbit,

$$\frac{d^2r}{ds} - \frac{dr}{ds} = 0$$

$$r\left(1 - \frac{2eE}{r}\right)\left(\frac{d\theta}{ds}\right)^2 = \frac{eE}{r}\left(1 - \frac{2eE}{r}\right)\left(\frac{dt}{ds}\right)^2$$

$$\frac{d^2\theta}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds} = G^3 g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)^2 \frac{d\theta}{ds}$$

and

$$\frac{d^2t}{ds^2} + \frac{d}{dr}\log\left(1 - \frac{2eE}{r}\right)\frac{dr}{ds}\frac{dt}{ds} = G^4 g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)^2 \frac{dt}{ds},$$

the values of the $g$'s taken from the form of the energy function $H$ being substituted in the expression for the $\Gamma$'s.

The solution of the last two equations is

$$p_3 = r^2\left(\frac{d\theta}{ds}\right) = p^0{}_3 e^{\int G^3 g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)dx^\alpha} \tag{4.6}$$

and

$$p_4 = p_4{}^0 e^{\int G^4 g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)dx^\alpha}$$

the $p_3{}^0$ and $p_4{}^0$ being constants of integration.

If we take the solution of the continuity equation of the form

$$G^3 = \text{constant and } G^4 = \text{constant},$$

then

$$p_3 = P_3 e^{\int g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)dx^\alpha}$$

$$p_4 = P_4 e^{\int g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)dx^\alpha} \tag{4.7}$$

and

$$-H = r^2\left(\frac{d\theta}{ds}\right)^2 - \left(1 - \frac{2eE}{r}\right)\left(\frac{dt}{ds}\right)^2 = H^2{}_0 \; e^{2\int g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)dx^\alpha},$$

where $H^2{}_0$ is the energy constant.

The above expression for the energy gives a type of energy function which fluctuates between the limits zero and $H^2{}_0$ along any path. However, in case the following condition is satisfied,

$$\sum_i \int g_{ii}\left(\frac{dx^i}{ds}\right)dx^i = 0,$$

summed on $i$ from 1 to 4, the energy function $H$ has the definite value $H^2{}_0$ along any particular orbit. In passing from one orbit to another, the value of the integral will increase (or diminish)

by a finite amount, say $\delta$, so that if the value of the energy along path I is

$$H_{\mathrm{I}} = H_{\mathrm{oI}}^2 \, e^{2\int g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)dx^\alpha} = H_{\mathrm{oI}}^2$$

when the above condition is fulfilled, then along path II

$$H_{\mathrm{II}} = H_{\mathrm{oI}}^2 \, e^{\int g_{\alpha\alpha}\left(\frac{dx^\alpha}{ds}\right)dx^\alpha + \delta} = H_{\mathrm{oI}}^2 \, e^{\delta} = H_{\mathrm{oII}}^2$$

The condition which we have imposed on the periodicity integrals is equivalent to the following,

$$\sum_i n_i = 0$$

$n_i$ = quantum number with regard to the $i^{\text{th}}$ vibration.

We are now in a position to determine the value of the frequency from the expression for the electromagnetic momentum (4·5),

$$\nu = \frac{k}{c}\left[\frac{p'^2_4}{g'_{44}} - \frac{p''^2_4}{g''_{44}}\right]$$

Substituting the value of $p_4$ into the above expression for $\nu$, namely,

$$p_4 = \frac{P_4}{H_0}\sqrt{H},$$

we obtain

$$\nu = \frac{k}{c}\left[\left(\frac{P^2_4}{H^2_0 g_{44}}\right)' H' - \left(\frac{P^2_4}{H^2_0 g_{44}}\right)'' H''\right]$$

Now from the first of the equations of motion,

$$n\left(\frac{d\theta}{ds}\right)^2 = \frac{eE}{r}\left(\frac{dt}{ds}\right)^2,$$

we obtain

$$\frac{(p_3)^2}{r^2} = \frac{eE}{r^2}\left(\frac{p_4}{g_{44}}\right)^2,$$

which gives

$$\frac{P^2_3}{r} = eE\frac{P^2_4}{g^2_{44}};$$

while from the energy equation, there results

$$\frac{P^2_3}{r^2} - \frac{P^2_4}{g_{44}} = -H^2_0.$$

Substituting and simplifying, we obtain

$$\frac{P^4_4}{H^2 g_{44}} = \left(1 - \frac{eE}{rg_{44}}\right) = 1 + \delta,$$

where $\delta$ is negligibly small compared with unity, so that

$$\nu = \frac{k}{c}[H' - H'']$$

+ terms in higher powers of $H'$ and $H''$.

The frequency of the light emitted by a radiating electron in moving from one orbit to another is given by the relationship

$$h\nu = H' - H'', \tag{4.8}$$

where $h$, whose value is $\frac{c}{k}$, is a universal constant.

Since the flux of energy along the various vectors is zero for the complete wave surface, the wave must be propagated equally in all directions from the origin—positive and negative—so that the wave front is a completely closed surface in the spatial three-dimensional manifold.

Although the equations of motion in the above form are not separable, it is possible by a change in the coördinate system to find a system of coördinates in which this is possible. In making the change, the equations of motion—equations of geodesics in Riemann space—are transformed invariantly, so that it is legitimate to consider the Riemann geodesic equations as representing the equations of motion of the electron whenever it is possible to make a transformation of coördinates in which the variables are separable.

### THE ACTIVITY OF THE FORCES.

The consideration of the rate of working of the field forces on the charged body, or, more aptly expressed, the activity of the forces of the electromagnetic field, leads to interesting results concerning the energy flow in the field.

The components of the ponderomotive forces acting on the field are given by (3.18).

$$P^i = \bar{G}^i \frac{dx^i}{ds} g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds}$$

from the equation of motion which we have derived.

But since the electromagnetic forces $F^{ij}$ assume the form

$$F^{ij} = \frac{dx^i}{ds} \frac{dx^j}{ds}$$

after the contact transformation has been effected (3.13), the derivates being the values of these functions for the charged body, since the $F$'s refer only to the electromagnetic intensities within the surface of the body.

Thus

$$P^i = \bar{G}^i F^{i\alpha} g_{\alpha\alpha} \left( \frac{dx^\alpha}{ds} \right), \tag{5.1}$$

if we limit the discussion to the case for which

$$g_{ij} = 0 \qquad i \neq j .$$

The expressions for the ponderomotive forces which we obtain in this manner are analogous to those assumed by Lorentz in his treatment of the optical properties of matter.

The activity of the forces $A$ on the charge in the field being the product of the ponderomotive forces $P^i$ by the spatial component of the velocity $V_i$ is, therefore, given by the relation

$$A = P^i V_i ,$$

$i$ summed from 1 to 3.

Since in any measuring system

$$P^\alpha g_{\alpha\alpha} \left( \frac{dx^\alpha}{ds} \right) = 0 ,$$

we obtain

$$P^i g_{ii} \left( \frac{dx^i}{ds} \right) = P^4 g_{44} \left( \frac{dx^4}{ds} \right),$$

summed on $i$ from 1 to 3; and the equation for the rate of working of the electromagnetic forces reduces to

$$A = \iiint P_4 g_{44} \left( \frac{dx^4}{ds} \right) dr, \tag{5.2}$$

integrated over the spatial dimensions of the charge.

If we choose the coördinate system, at the time $t$ we are considering, in such a way that

$$g_{jj} \left( \frac{dx^j}{ds} \right) = V_j \text{ and } g_{ii} \left( \frac{dx^i}{ds} \right) = 0 \qquad i \neq j$$
$$i = 1, 2, \text{ or } 3.$$

and call the electric intensity in the direction of the resultant spatial velocity $V$, $E$, then

$$P^4 = \bar{G}^4 F^{ij} g_{jj} \left( \frac{dx^j}{ds} \right) = \bar{G}^4 V E$$

so that we finally obtain for the activity

$$A = \iiint (\bar{G}^4 VE)\left(g_{44}\frac{dx^4}{ds}\right)dr.$$

Now from the second set of Maxwell's equations

$$\bar{G}^4 . V = (F_j^{\alpha})_{\alpha}$$

whence, substituting into the above equation, we obtain

$$A = \iiint (F_j^{\alpha})_{\alpha} F^{4j}\left(g_{44}\frac{dx^4}{ds}\right)dr$$

for any definite value of $j$, or in general

$$A = \iiint (F_{\beta}^{\alpha})_{\alpha} F^{4\beta}\left(g_{44}\frac{dx^4}{ds}\right)dr. \tag{5.3}$$

Since for the motion we have been considering $g_{44}\frac{dx^4}{ds}$ = constant along any path at any instant,

$$A = g_{44}\left(\frac{dx^4}{ds}\right)\iiint (F_{\beta}^{\alpha})_{\alpha} F^{4\beta}dr.$$

Making use of the generalization of Stoke's theorem to any surface immersed in a higher order manifold,[11] we obtain

$$A = \iint g_{44}\left(\frac{dx^4}{ds}\right) F^{4j}(F_{ji}g^{ji})\,dx^i dx^{\kappa}$$

+ volume integral,

$$= \iint g_{44}\left(\frac{dx^4}{ds}\right) F^{4i}F^j_i dx^i dx^{\kappa} \tag{5.4}$$

+ volume integral, where all the various permutations of $i$, $j$ and $\kappa$ are taken for which $i \neq j \neq \kappa$.

The only interpretation consistent with the validity of the conservation of energy is that the surface integral represents the flux of energy into the region $r$ from the outside. If we apply the above solution for the activity to the case of the Special Relativity Theory of the slowly moving electron, we have

$$g_{44} = -1$$

and $\frac{dx^4}{ds} = \frac{dt}{ds}$ = unity approximately, so that

$$A = -\iint [E \cdot H]_n\,ds + \text{volume integral over region of } \gamma \tag{5.5}$$

where $(E.H)_n$ = the vector product of the electric intensity $E$ and the magnetic $H$ along the normal to the surface; and where

[11] SCHOUTEN, "Der Ricci Kalkul," p. 97.

the difference in units is due to the fact that we have taken the velocity of light as unity.

In this case, then, the surface integral becomes identical with the Poynting energy flux.

If we consider the value of the integral

$$-\int\int F^{4i} F^{j}_{i} dx^{j} dx^{\kappa}$$

integrated over the infinite boundary of the spatial manifold, we shall always obtain for this expression a value which corresponds to the continuous radiation of energy from the system, independent of the type of motion considered. However, when we consider the complete expression for the radiation, namely,

$$\int\int g_{44}\left(\frac{dx^4}{ds}\right) F^{4i} F^{j}_{i} dx^{j} dx^{\kappa},$$

the sign of the energy flux, whether representing absorption or emission, will depend on the sign of the factor

$$g_{44}\frac{dx^4}{ds}.$$

In the type of periodic motion we have been considering in this paper, the value of this expression over a complete cycle fluctuates between the values $P_4$ and $-P_4$ for any given orbit, the average value of the expression being zero. In this type of motion, then, the amount of energy radiated while the electron completes a cycle in any given orbit is zero, so that radiation, or absorption, of energy occurs only when the electron executes a non-periodic motion.

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**Interpretation of Deviations from Ohm's Law.** KARL T. COMPTON. (*Proc. Nat. Acad. Sci.*, Sept., 1926.)—Five years ago Bridgman published results of experiments showing departures from Ohm's Law amounting to a few per cent. when thin films of gold or silver are traversed by currents so considerable in proportion to the cross-section that the current density amounted to several million amperes per sq. cm. The author uses this result to test the correctness of two common theories of metallic conduction. Thomson's Doublet Theory can account for the departure from Ohm's Law only by requiring an impossible assumption. The Free Electron Theory, on the other hand, furnishes a rational explanation of the departure. This theory regards metals as containing free electrons that are acted on by the electric field during their free existence and accordingly move so as to cause a net transfer of electricity in one direction.

G. F. S.

# A THERMOSCOPE FOR LIQUID-AIR BATHS.*

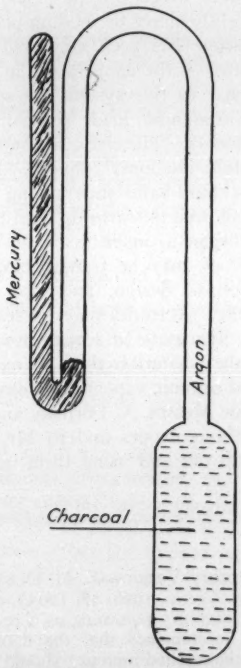
BY

**ENOCH KARRER, Ph.D.**

The Cleveland Wire Works, General Electric Company;
Member of the Institute.

A SIMPLE device for observing and controlling low-temperature baths makes use of the absorption properties of

FIG. 1.




charcoal at low temperature. A tube partially filled with granular charcoal and a gas is connected to a manometer (a simple U-tube

* Communicated by the Author.

with mercury). The mercury surfaces will be displaced depending upon the amount of the gas that is held by the charcoal; that is, upon the temperature of the charcoal. One may choose the range of greatest sensitivity by properly selecting the gas that is in contact with the charcoal.

A specific case is sketched in Fig. 1, where argon is used in the charcoal chamber. The pressure of the argon is about 20 cm. when the charcoal is at room temperature. The space above the mercury in the other limb of the manometer is evacuated. The temperature range over which this particular thermoscope indicated is near and slightly above the boiling-point of argon. This argon-filled thermoscope has been employed over an extended interval for the control of the temperature in a purifying system for helium and neon. In this system the space containing the purifying charcoal was annular, made by welding together concentrically two steel cylinders. The charcoal chamber of the thermoscope was inserted into the inner cylinder. During the routine manipulation the liquid-aid baths surrounding the purifying charcoal chambers were allowed to warm up until the mercury surface in the thermoscope began to move.

Such a thermoscope may be transformed into a real thermometer by attention to design, and also into an automatic controlling device since electrodes may be sealed in and a relay operated through it. It may be of service in other lines than that mentioned, where other materials than charcoal and argon with selective porosity and organic vapors are concerned.

My thanks are due Messrs. A. Poritsky and John Birdsell for making and filling thermoscopes and to Mr. W. Eppinger for faithfully trying them out and using them in the production of helium and neon.

CLEVELAND, OHIO,
May 25, 1926.

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**Poisoning by Mercury Vapor.**—L. M. DENNIS, of Cornell University (*Ind. and Eng. Chem.*, 1926, **18**, 1205), calls attention to the occurrence of chronic mercury poisoning as a result of inhalation of mercury vapor. He recommends that the floor of the laboratory be free from cracks, that spilled mercury should not be permitted to lie in the room, that all containers of mercury be covered or stoppered when not in use, that ventilation be efficient and thorough, and that work with open mercury be conducted in a hood whenever possible. J. S. H.

# FLOW OF LIQUIDS IN PIPES OR CHANNELS.*

BY

H. W. MARCH and C. N. WARD.

A PAPER entitled "A New Theory of Fluid Flow" [1] has just come to our attention. This paper purports to give a new theory of fluid flow based upon a mathematical proof.[2] In the present note we wish to call attention to the fact that the proof given is not compelling, as it is based not upon dynamical reasoning, but upon a merely descriptive consideration of the variation of the velocity. Furthermore, the results of this theory are at variance with results which are well established both theoretically and experimentally.

The proof referred to is found on page 773. Liquid is flowing in a straight wide channel of rectangular cross-section. $y$ is the distance from the bottom of the channel, and $x$ is the velocity in the direction of the length of the channel. As a result of a qualitative discussion of the velocity distribution, the substance of which is that the velocity-gradient, $\frac{dx}{dy}$, diminishes as $y$ increases, the author concludes that $\frac{dx}{dy}$ is inversely proportional to $y$. An infinite number of laws of variation of $\frac{dx}{dy}$ with $y$ can be assumed, each expressing the fact that $\frac{dx}{dy}$ decreases as $y$ increases. No reason, dynamical or otherwise, is adduced for making the particular assumption that $\frac{dx}{dy}$ is inversely proportional to $y$.

The author then states that the same reasoning applies to straight pipes of circular cross-section, the wall replacing the bottom of the channel and the axis replacing the surface of the water. Now in the case of stream-line flow in a long straight circular tube, it follows from the hydrodynamical equations of motion of a viscous fluid, that the velocity, $x$, is given by the formula [3]

$$x = \frac{p_1 - p_2}{4\mu l}(a^2 - r^2) = C(a^2 - r^2), \qquad (1$$

* Communicated by the Authors.

[1] Anthony Bruce Cox, JOUR. FRANK. INST., Dec., 1924.

[2] *L. c.*, p. 771 and p. 773.

[3] Lamb, "Hydrodynamics," 3rd Edition, p. 544.

where $C$ is a constant, $a$ is the radius of the tube, and $r$ is the distance from the axis. From this equation $\frac{dx}{dr} = -2Cr$.

Or writing $r = a - y$, where $y$ is the distance from the wall,

$$\frac{dx}{dy} = 2C(a - y). \tag{2}$$

The relation (1) has been known for many years. Large numbers of experiments have been performed[4] in which this relation or conclusions from it have been verified. We are not aware of the slightest experimental evidence that it is not valid for stream-line motion. Formula (2) shows a decrease of the velocity-gradient, $\frac{dx}{dy}$, with increasing $y$, but it is not the law

$$\frac{dx}{dy} = \frac{k}{y}, \tag{3}$$

assumed by Mr. Cox as the fundamental law upon which his theory for all types of flow, stream line or turbulent, is based.

Mr. Cox's law

$$y = ae^{bx}, \tag{4}$$

derived from (3), compels him to draw the velocity-distribution curves for a circular pipe, with a sharp "nose" at the centre. Since the experimental points in the figures given by the author are fitted as well or better by smooth curves without this "nose," it is unnecessary to discuss further this obviously impossible dynamical situation.

In conclusion, it should be pointed out specifically that in the case of turbulent motion, the velocity distribution curves are not parabolas and that smooth curves can be drawn which fit the experimental data much better than some of the dotted curves drawn for comparison in Mr. Cox's paper.

### REJOINDER BY MR. COX.

Referring to your letter of September 7th and to my letter of September 20th, I wish to comment on Professors March and Ward's criticism of my fluid flow paper as follows. The comment is as brief as I can make it and still reply satisfactorily to the questions raised.

[4] See *e.g.* Poseuille, "Mém. des Sav. Étrangers," t. IX, 1846. O. Reynolds, *Phil. Trans.*, **174**, p. 935, 1883.

COMMENT.

Professors March and Ward have been unlucky in their choice of material in my fluid flow theory to criticize. This is no doubt due largely to their not being familiar with the history of the development and publication of the theory.

Although the theory as published apparently has as a part of its basis a mathematical "proof," as a matter of actual recorded fact the entire theory was developed only on the basis of the experimental data then available, and the mathematical proof was added afterward because one of the earlier reviewers of the paper thought that there ought to be one. It was placed in the early part of the paper because there was no other place where it would fit in well. However, this proof does have a correct experimental basis, as will appear later.

The second error was made by Professors March and Ward because, on account of the great length of the original manuscript of my paper, a large amount of the text, as well as many of the curves, were omitted from the published paper. Among the parts deleted was a Supplement which exposed the fallacy of the "proof" of the equations in Horace Lamb's "Hydrodynamics," which are cited by Professors March and Ward. Lamb, in his proof of these equations, refers to Article 30, page 30, of his "Hydrodynamics" (4th ed.), where he has made the same false assumption that is made by so many writers on this subject. Since Lamb's book is full of abstruse mathematical and scientific terms which are difficult for most of us ordinary mortals to understand, I shall preferably refer to a similar but simpler proof in Archbutt and Deely's "Lubrication and Lubricants." I quote from the Supplement to my original manuscript:

"Archbutt and Deely assume for their proof of the parabolic form of the velocity-distribution curve that the rate of shear in a fluid between two parallel plates, one plate moving and the other stationary, is constant. They use a pack of cards to illustrate the manner in which the layers of fluid move over each other when the fluid is flowing at a rate less than the critical, claiming that if a deck of cards be placed on the table and pressure applied to the top card simultaneously with a tangential force, all the cards will be uniformly and proportionately displaced. Actual trial of this experiment with cards, however, shows that the upper cards are much further displaced than the theory would indicate

and the lower cards much less. This appears reasonable to me since all the energy to move all the remaining layers below has to be transmitted from the top layer to the next one below, each layer next below transmitting less and less energy as the stationary bottom layer is approached. But in order that the layers may transmit these relatively different amounts of energy, the difference in velocity between the upper layers must be relatively greater than between the lower layers. This statement is necessarily true in the case of fluid flow also, regardless of the manner in which the coefficient of fluid friction may vary with respect to the velocity." The basic assumption for the equations cited by Professors March and Ward is therefore false, and the equations resulting therefrom must be discarded.

A third error which these gentlemen have made is that of referring to Lamb's work as authoritative at the same time they criticize me for making an assumption without fully stating the basis therefor. Quoting from the fourth edition of Lamb's "Hydrodynamics" (the 3rd ed. is not available to me), I find on page 569, article 326, "The simplest hypothesis we can frame is that these functions are linear." And farther on in the same paragraph, ". . . this being the most general assumption consistent with the above suppositions, and with symmetry." And on page 572, article 327, ". . . the most natural supposition to make is . . ." In article 327, page 572, and again in article 331, page 578, Lamb has made the same fundamental assumption which I have made in my paper, namely, that there is no slip of the fluid at the pipe wall, or as he further explains, that there is a stationary layer of fluid which clings to the pipe wall. I wish here to repeat the statement on page 769 of my own paper that, "The only curve known to me which will represent the condition of the fluid across the entire diameter of the pipe, and yet account for the existence of this stationary film, is the exponential curve . . . $y = ae^{bx}$ . . ."

Referring to my critics' claim that round-nosed curves can be drawn which will better represent the flow data available, I shall await the publication of such curves with very great interest. If families of such curves can be drawn, the result can only be a more nearly correct theory of flow than the one which I have originated. The constants of such curves, like those of the families which I have drawn, are bound to show a relation which

must have a physical meaning, and the values of these constants must necessarily depend on the physical characteristics of the fluid and of the pipe, as I have also tried to bring out in my paper.

In conclusion, it seems to me that these critics must have misjudged my attitude else they would have made their criticism a little less sweeping as well as perhaps a little more temperate. Since this is the first time any one has developed a comprehensive theory of the flow of fluids which will stand the direct comparison with actual flow data which I have given it, I am compelled to believe that the new theory is not *wholly* without scientific value. However, for fear that these critics might continue to misjudge my attitude, I again quote from some of the parts deleted from the original manuscript. "It must, of course, be admitted at the outset that the flow problem is a very large and complex one, and that therefore it is not to be expected that the new theory will at once and without modification fully and completely explain every type and condition of flow. . . . Nevertheless, from the fairly wide range of phenomena the theory is hereinafter shown to cover successfully under simple 'ideal' flow conditions, it appears to afford a solid basis for building a practical theory which may be satisfactorily applied in many cases to conditions of flow other than ideal—that is, commercial conditions. If such a theory is ever to be built, it would seem to be necessary to approach it in some such fashion as that outlined in this paper."

Yours truly,

ANTHONY BRUCE COX.

September 25, 1926.

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**The Effect of Acetone as an Impurity in Spirit Thermometers.** W. F. HIGGINS. (*Jour. Scientific Instruments,* Aug., 1926.)—In the National Physical Laboratory, Great Britain, it had been noted that certain spirit thermometers showed a marked fall in their readings when exposed to the same temperature after the lapse of years. This could not be attributed to changes in the glass of the instrument because that is known to contract. This would force the liquid up the stem and thus cause a higher reading. Besides this, volume changes in the glass, while of importance with mercury thermometers, become insignificant with spirit instruments on account of the large coefficient of expansion of this liquid. A search for minute cracks through which the alcohol might escape was unsuccessful. The

placing of the thermometer in a vacuum failed to produce any acceleration of the fall of the liquid, due to the more ready escape of the spirit through the suppositious cracks. Since, therefore, it appeared that the glass was not responsible for the fall of readings, the cause must reside in the liquid itself.

Acetone is a common impurity in methyl alcohol and is known to form condensation products under the influence of light, while neither pure ethyl nor pure methyl alcohol is suspected of changing volume with the passage of time. Experimental thermometers were prepared, having ten or twelve degrees per inch. Readings of the ice-point were taken at intervals of two or three days. Instruments with pure ethyl or methyl alcohol showed no variation of reading, no matter whether the residual gas above the liquid was air or nitrogen. On the contrary, thermometers with 50 per cent. acetone and 50 per cent. methyl alcohol changed greatly and for a long time. Thus after 480 days the reading of the ice-point, originally 32° F., had fallen to −30°, when the residual gas was air and to −33° when it was nitrogen. Moreover, the rate of change depended on the time of year, being least in the dark winter months and most rapid in the summer with longest days and maximum illumination.

As soon as acetone was detected as a disturber of the reliability of thermometers, additional instruments were made and put to test, the manufacturer being asked to keep the solutions and finished products in the dark as far as possible. After these new thermometers had been under observation for six months, definite features in their behavior were established. When the liquid used is pure ethyl or pure methyl alcohol or a mixture of the two, no change of reading occurs with the passage of time. The addition of red aniline coloring matter makes no difference in the comportment of ethyl alcohol. Thermometers containing equal parts of ethyl alcohol and acetone show no change in the ice-point so long as they are kept in the dark. Exactly similar instruments exposed to the light suffer changes in reading comparable to those of the two thermometers above cited. "Exposure to light therefore plays an important part in the effect noted and this result strongly supports the theory already advanced that change of reading is due to polymerization of the acetone present under the influence of light."

A thermometer containing a liquid with 1 per cent. acetone showed a reduction of its ice-point amounting to 3.2° F. in 200 days. When the content was 2 per cent. the lowering in the same time was 5° and for 10 per cent. it was 13.5°. Two thermometers filled with commercial methylated spirit and with redistilled methylated spirit, respectively, lost 2° each in the course of 140 days. It thus appears that reliable thermometers can be made with the use of pure methyl or ethyl alcohol or of a mixture of them, but that acetone must be banned from thermometric use as must be methylated spirit, commercial or distilled, because of its acetone content. G. F. S.

## NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

### SIMPLIFIED DEDUCTION OF THE DOPPLER-EFFECT FORMULA FOR ELLIPTIC SPACE.[1]

By L. Silberstein.

It is shown that the previous deduction (*Phil. Mag.*, 48 : 619, 1924) of the Doppler-effect formula for any inertial motion of the star and the observing station in the de Sitter space-time is considerably simplified by making use at the outset of the first integral, $\cos^2\sigma.cdt/ds=\gamma=$const., of the world-geodesic equations. This gives the element $ds$ of the observer's proper time, which with the corresponding element of the star's proper time, as used in the original deduction, leads at once to the required rigorous formula. Whence, by developing up to second-order terms, follows the approximate formula for the spectroscopic radial velocity $V$ in terms of the star's distance, divided by the curvature radius $R$ of space-time, and certain two constants characterizing the individual star.

In the second part of the paper the applications of this formula are considered. It is shown that the absolute value of the radial velocity $/V/$ can only be expected to increase on the whole with the star's distance, $r$, this correlation being masked by the haphazard distribution of the said two star-constants among the celestial objects. Whence the comparatively low value of the correlation coefficient between $/V/$ and $r$, namely, $0.320 \pm 0.135$ for the available twenty objects, comprising eighteen globular clusters and the two Magellanic Clouds. If two of the globular clusters are discarded, the correlation coefficient for the remaining eighteen objects is $0.636 \pm 0.095$, which is about seven times its probable error. This seems to make the existence of the predicted correlation and a curvature radius $R=6$ to $7.10^{12}$ astronomical units tolerably likely, such a value of the radius being yielded statistically by the last-mentioned group of objects.

* Communicated by the Director.

[1] Communication No. 260 from the Research Laboratories of the Eastman Kodak Company and published in *Popular Astronomy*, 34 : 246, 1926.

## ON THE RELATION BETWEEN TIME AND INTENSITY IN PHOTOGRAPHIC EXPOSURE. IV.[2]

By L. A. Jones and V. C. Hall.

FURTHER results of the study of the reciprocal relation between the time of exposure and the intensity of exposing radiation are given. It has been found that the maximum density obtainable with complete development is dependent upon the intensity used in making the exposure. This indicates that for the silver halide grains there is an intensity threshold below which developability cannot be produced, no matter how long the exposure time is prolonged. It has also been found that if the exposing intensity be sufficiently high, all of the silver halide present in the emulsion is made developable.

## DISPLAY ENLARGEMENTS FOR SINGLE FRAME MOTION PICTURES.[3]

By K. C. D. Hickman.

THE highest quality enlargement obtainable from a single motion picture "frame" cannot be secured unless the following precautions in descending order of importance are taken:

The light source must be highly diffused. The last diffusion screen must be as near as possible to the negative. The negative must be placed between glasses quite free from defects and dust. The negative may be embedded in a liquid of the same mean refractive index.

The paper described methods of sandwiching the film with glycerine or xylol between glasses before placing in the projector.

## NOTE ON THE STRENGTH OF SPLICES.[4]

By S. E. Sheppard and S. S. Sweet.

PROPER film splicing should be done automatically. It was found that there is no difference in strength between the 0.156-inch and the 0.078-inch splice. The use of a proper scraper with

[2] Communication No. 266 from the Research Laboratories of the Eastman Kodak Company and published in *J. Opt. Soc. Amer.*, 13, 443, 1926.

[3] Communication No. 275 from the Research Laboratories of the Eastman Kodak Company and published in *Trans. Soc. Mot. Pict. Eng.*, No. 25, 49, 1926.

[4] Communication No. 276 from the Research Laboratory of the Eastman Kodak Company and published in *Trans. Soc. Mot. Pict. Eng.*, No. 25, 1926, p. 142.

a square edge is recommended. Time increases the strength of splices, but this effect may be replaced largely by heat. Excess use of cement should be avoided, as it causes buckling and warping.

## EFFECT OF PROJECTION LENS FLARE UPON THE CONTRAST OF A MOTION PICTURE IMAGE.[5]

By L. A. Jones and C. Tuttle.

A STUDY is made of the relative characteristics of a motion picture positive and its screen image. It is concluded that: (1) The veiling brightness due to flare is directly proportional to the average transmission of the projected positive. (2) The flare-forming characteristic of a lens may be expressed as the ratio of the flare brightness to the average brightness of the picture. (3) The effect of flare upon the contrast characteristics of the picture can be computed from a knowledge of certain characteristics of the positive. (4) The effect of lens flare on quality of tone reproduction is to warp the shape of the reproduction curve and depress the contrast.

**The Variations of Pressure with Temperature in Evacuated Vessels.** N. R. CAMPBELL. (*Phil. Mag.*, Aug., 1926.)—In 1919 J. E. Shrader published some interesting observations on the pressure changes that take place in glass vessels sealed off when the pressure within them is low and subsequently heated. He found that a vessel of this material, baked at T° while the pressure was being reduced to a low value and then sealed off and cooled down, would have a pressure greater than that at the time of sealing when it was heated to a temperature below T°. In the present series of experiments, conducted in the laboratory of the General Electric Company, Wembley, glass vessels, thoroughly exhausted at a temperature of at least 380° C., were again and again subjected to these three operations: (*a*) Heating to some temperature between 50° and 200°; (*b*) cooling to room temperature; (*c*) passage of a suitable electric discharge through the gas with resulting reduction of pressure. The effect of the discharge was to cause the pressure at the end of any operation *a* to be less than at the close of the preceding one. The same is true for operation *b* also. However, in both cases the pressures seem to approach asymptotically a final value, that is governed by the higher

[5] Communication No. 277 from the Research Laboratory of the Eastman Kodak Company and published in *Trans. Soc. Mot. Pict. Eng.*, No. 25, 153–163 (1926).

temperature of the cycle. When this is 120° C. the limiting value is of the order of .00001 mm. of mercury.

The deposition upon the glass walls of the vessel of layers of nickel, tungsten or molybdenum brought about the disappearance of the progressive reduction of pressures noted in the absence of a metal coating. Silver, copper, magnesium and zinc failed to work in the same way as the three metals above named. In all the pressure changes investigated it seems that $H_2O$ and $CO_2$ are the chief constituents of the residual gas involved. The experiments do not, however, solve the problem of the manner in which an electric discharge in a gas promotes the adsorption of the gas and the resulting lowering of pressure. For high-vacuum technique, on the other hand, there is a valuable outcome. "Our observations indicate that it is impossible to make a vessel with bare glass walls in which the pressure will remain permanently as low as $10^{-6}$ mm. in such conditions as are likely to prevail in practice. On the other hand, by coating the walls with evaporated metal, and taking care that the pressure during the evaporation is as low as possible, it is quite easy to obtain vessels in which the pressure at 20° C. is permanently less than $10^{-7}$ mm. and not more than $10^{-6}$ mm. at 120° C."

G. F. S.

**Allotropy of Germanic Oxide.**—Germanic oxide (germanium dioxide) exists in several allotropic modifications, including two soluble varieties, an insoluble or alpha modification, a metastable glass, and a devitrified glass. The soluble varieties are partially converted into the insoluble modification by heating at temperatures between 225° and 1100° C.; this modification is converted into the glass on heating above the melting-point (1100° C.); the glass spontaneously devitrifies with the passage of time. JOHN HUGHES MÜLLER, of the University of Pennsylvania (*Proc. Am. Phil. Soc.*, 1926, **65**, 183–195), has made an extensive study of these modifications. The *insoluble form* is remarkably inert toward reagents. Its formation is independent of the source of the soluble oxide from which it is prepared. It is about 1000 times less soluble than the soluble forms; its solubility at the boiling-point of water is approximately double its solubility at 25° C. One hundred grams of water dissolve $0.00045 \pm 0.00002$ gram at 25° C., and $0.00117 \pm 0.00002$ gram at 100° C. Its density is 6.0039 at 24° C. Both *glassy forms* (metastable and devitrified) are completely soluble in water. The metastable glass has a density of 3.3037 at 24° C. The *soluble forms*, obtained by evaporation and by hydrolysis, have densities at 24° C. of 3.5205 and 3.6128, respectively. The X-ray spectrographs of the various modifications show that the insoluble form has a totally different diffraction pattern from any of the soluble forms.

J. S. H.

## NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

### STUDIES IN VAPOR PRESSURE. II. MONONITROTOLUENES.[1]

By J. F. T. Berliner and Orville E. May.

[ABSTRACT.]

THE vapor pressures of the three isomeric mononitrotoluenes have been determined from 50° to a few degrees above their boiling-points. The boiling-points at atmospheric pressure have been found to be as follows: *o*-nitrotoluene, 220.38°; *m*-nitrotoluene, 231.87°; *p*-nitrotoluene, 238.34°.

The latent-heat equations have been derived and the latent heats of vaporization calculated. The pressures calculated from these equations agree well with the observed values.

The entropies of vaporization at a concentration of 0.00507 mole per litre indicate that the molten mononitrotoluenes are normal liquids.

### NON-VOLATILE CONSTITUENTS OF THE COTTON PLANT.[2]

By Frederick B. Power and Victor K. Chesnut.

[ABSTRACT.]

GROUND branches of the cotton plant, from which the coarse twigs and bolls had been separated, were extracted with alcohol. The alcohol, essential oils, acetic and formic acids, and furfural, together with ammonia and trimethylamine, presumably from the choline present, were driven off by evaporation and steam distillation.

The aqueous liquid and the black, oily resin left after distillation were examined. From the former potassium nitrate, potassium chloride, quercetin, betaine, choline, succinic acid, and a reducing sugar were obtained, together with a very small quantity of a neutral crystalline substance melting at 140–143° and a crystalline acid melting at 105–106°. The presence of salicylic acid was also indicated. The oily resin represented 3.13 per cent. of the dry material employed. From this, after extraction with various solvents and subsequent hydrolysis of some of the extracts

* Communicated by the Chief of the Bureau.

[1] Published in *J. Am. Chem. Soc.*, **48**, October, 1926, No. 10, pp. 2630–2634.

[2] Published in *J. Am. Chem. Soc.*, **48**, October, 1926, No. 10, pp. 2721–2737.

and vacuum distillation of others, the following substances were isolated: A phytosterol, a phytosterolin, glucose, pentatriacontane, palmitic acid, an acid of phenolic character, melting at 188–189°, butyric, valeric, and caproic acids, a small quantity of a phenolic substance, dipentene, an oxygenated compound, boiling at 200–220°, and an optically active, dicyclic sesquiterpene. The presence of a product, boiling at 300–350°, consisting of diterpenes and a small quantity of a phenolic substance, was indicated.

## OPTICAL ROTATION AND ATOMIC DIMENSION. VI.[3]

**By D. H. Brauns.**

[ABSTRACT.]

THE specific-rotational values reported in the literature for chloro-, bromo- and iodo-hepta-acetyl cellociose have been revised for comparison with the fluoro-hepta-acetyl compound values. In addition to the fluoro-hexa-acetyl and octo-acetyl compounds, the fluoro-, chloro-, bromo- and iodo-hepta-acetyl derivatives of 4-glucoside-mannose have been prepared and described. A comparison of the specific rotational values shows that these two sets of *biose* derivatives deviate from the regular relationship observed for the corresponding derivatives of the *monose* sugars. An agreement is, however, obtained by excluding the values for the fluoro derivatives. When studied for other bioses these deviations may lead to a more detailed knowledge of the spatial arrangement of the atoms in the bioses.

## ACETYL GROUPS IN PECTIN.[4]

**By E. K. Nelson.**

[ABSTRACT.]

THE acetyl group, while evidently an essential constituent of beet pectin, is not an essential constituent of pectins in general.

Root pectins, such as beet pectin, may be related to or derived from lignin, ligno-cellulose or cell materials containing acetyl groups in their molecules. Fruit pectins, so far as examined, differ from sugar-beet pectin in being practically devoid of acetyl groups.

[3] Published in *J. Am. Chem. Soc.*, **48**, November, 1926, No. 11, pp. 2776–2788.

[4] Published in *J. Am. Chem. Soc.*, **48**, November, 1926, No. 11, pp. 2945–2946.

## NOTES FROM THE U. S. BUREAU OF MINES.*

### AN X-RAY STUDY OF LIMES OF DIFFERENT PLASTICITY.

**By Marie Farnsworth.**

WORK was completed on an X-ray study of limes made by burning marble and precipitated calcium carbonate in air at 1800°, 2000° and 2200° F., and marble in a vacuum furnace at 1200° to 2400° F., in steps of 200°. The plasticity of all the samples was measured and X-ray powder photographs taken. It was found that the lines of the pattern differ for samples of different plasticities, and thereby furnish an index of the plasticity.

The object of the work was to determine whether additional information could be obtained by this method, that would help to explain differences of plasticity shown by limes from limestones of the same chemical composition.

Details are given in a paper presented to the *Journal of Industrial and Engineering Chemistry*.

### ALKALI CYANIDES IN THE IRON BLAST-FURNACE.

**By S. P. Kinney and E. W. Guernsey.**

FOR a number of years the Bureau of Mines has been studying the physical and chemical reactions that take place during the smelting of iron in the blast-furnace. It has made experiments in the laboratories of the experiment stations at Pittsburgh, Penna., and Minneapolis, Minn., has made test runs with an experimental blast-furnace at Minneapolis, and has collected a large mass of data on the work done by commercial blast-furnaces.

The presence of alkali cyanides in the interior of the iron blast-furnace has been noted repeatedly during a period of almost 100 years, and has been studied more or less closely by a few persons.

In order to establish more definitely the alkali-cyanide concentration in the gases from various regions of the modern blast-furnace, the Bureau of Mines, in coöperation with the Fixed Nitrogen Laboratory, undertook an investigation in 1924 in conjunction with a more general study of the fundamental reactions

* Communicated by the Director.

occurring in the blast-furnace. In the course of that study, samples of gas and stock were taken across a series of planes at various heights in the furnace.

The intention is to extend this investigation ultimately to several blast-furnaces in order to note the variations in cyanide concentration with variations in charge and operating conditions. The measurements for one furnace have been completed. Technical Paper 390 gives the results and a résumé of previous work bearing on the subject.

## CARBON MONOXIDE RECORDER AND ALARM.

**By S. H. Katz.**

AT THE request of the New York State Bridge and Tunnel Commission, which desired information on the contamination of tunnel air by carbon monoxide in the exhaust gases of automobile engines, the Bureau of Mines made tests to obtain necessary data for a ventilating system for the Holland vehicular tunnels under the Hudson River at New York City, now being constructed. The data obtained for the Holland tunnels were applicable to the Liberty vehicular tunnels at Pittsburgh, Penna., completed in 1924.

Four parts of carbon monoxide in 10,000 parts of air were determined as the maximum concentration to which a person may be exposed for one hour without noticeable effects; or, if the concentration is increased from zero uniformly for one hour, the maximum concentration tolerable is six parts per 10,000, averaging three parts per hour. An apparatus that would automatically determine and record such low concentrations of carbon monoxide was developed by the Bureau of Mines.

The recorder is equipped with a bell to warn attendants in the fan house should the CO in its section at any time exceed four parts in 10,000. Further, the recorder can effect considerable saving in power at the fan house if the volume of air is reduced when the CO is low.

Carbon monoxide recorders should be useful wherever CO is an important factor because of hygienic, technical, or economic reasons, or in gaseous processes when CO can be introduced as an inert component. The recorder has proved to be the most sensitive instrument known for determining carbon monoxide. Details are given in Technical Paper 355, just issued by the Bureau.

## CHEMICAL ACTION OF AN ELECTRIC SPARK IN IGNITION OF METHANE.

By H. F. Coward and E. G. Meiter.

Principal Assistant, Safety in Mines Research Board, British Mines Department.

Assistant Physical Chemist, Bureau of Mines, U. S. Department of Commerce.

In the joint investigations being conducted for increasing safety in mines, by the U. S. Bureau of Mines and the Safety in Mines Research Board of the British Mines Department, the nature and sequence of the chemical changes that occur during the ignition of an explosive gas mixture are being studied by the writers. In the present experiments methane-air mixtures were chosen on account of the practical importance of a knowledge of the conditions affecting ignition of methane in mines.

The amount of reaction in the path of single electric sparks passed through explosive mixtures of methane and air, when the sparks are just too weak to cause ignition, were successfully measured, and analyses of the products have been obtained which give a picture of the chemical changes. The amount of reaction of a single spark was determined by passing uniform sparks at definite time intervals until the progress of reaction was measurable by gas analysis. Details of the experiments will be published in a paper presented to the *Journal of the American Chemical Society*.

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**Protection of Reservoirs.** (U. S. Department of Agriculture, Bulletin No. 1430-D.)—Suggestions that may add to the value of water-power and water-supply developments are made in a bulletin just issued by the Forest Service, entitled "The Financial Limitation in the Protection of Reservoirs," by W. W. Ashe. It is of interest to investors in water power and in municipal water supply securities as well as to managers of such properties and to engineers.

Storage of water is becoming a recognized factor in hydro-electric development, just as it is necessary in most cases for municipal supply. This bulletin shows that with storage there enters the problem of siltage through erosion of soil, an insidious agency which may cause loss of pondage and consequent reduction in the capacity of reservoirs.

The rapidity of erosion and of silting up of reservoirs is affected by different physical and meteorological conditions in the different regions of the United States. Over the greater portion of northern and northwestern United States and in Canada erosion is not the serious problem that it is in parts of the Southeast, and particularly in portions of the Southwest. The bulletin calls attention to the high silt burden of certain streams and the rapidity with which

storage capacity of reservoirs is reduced, as in the case of the Colorado River (of Texas). The reservoir at Austin on this river lost more than half its capacity within ten years. Many other streams carry a silt burden fully as great as that of the Colorado. On the basins of certain of these streams, according to Mr. Ashe, the surface conditions are such that erosion can be materially reduced. On others the conditions are such that erosion can be only slightly lessened. On these it is a natural condition, the result of concentrated rainfall on surfaces naked of protective vegetation on account of irregular rainfall. There are other regions, the bulletin points out, where protection of the surface is highly efficacious in reducing erosion.

It is shown that it is possible to ascertain the rate of silting which is taking place and from this to determine the capital value of the storage capacity, the loss of which might be expected within a given period. It is also possible to determine the extent to which excessive erosion may be checked by artificial means. Mr. Ashe proposes a formula for determining how much money could be economically expended to maintain this capital value, which otherwise would be destroyed by siltage. He points out that in case woods are artificially established for the purpose of securing soil cover and reducing erosion, a definite return can be expected from the investment therein, but that power and water companies are justified in making additional expenditures which would be within the capitalized value of the storage capacity which will be preserved.

In the Southern Appalachians as well as elsewhere there are alluvial lands which have been injured during floods by gullying or by the deposit of sand and gravel. Where such conditions exist there is, in the opinion of Mr. Ashe, an opportunity for the establishment of settling basins. Settling basins of the kind he proposes would not only result in the elimination of much silt from the stream, but also would bring about the rehabilitation of the devastated lands, which after being built up by deposit of the sediment would again be available for farming. Since this sediment as a rule consists largely of the most fertile soil eroded from the surface of the watershed, areas thus built up would be very productive.

**New Natural Base.**—S. B. SCHRYVER and H. W. BUSTON (*Proc. Royal Soc.*, B, 1926, **100**, 360–367) have isolated protoctine, a hitherto unknown base, from the products of hydrolysis of the proteins of oats and castor beans. It has the empirical formula $C_8H_{15}N_3O_5$, contains one amino group, one hydroxyl group, one carboxyl group, and no alkyl groups, apparently contains a basic group similar to the iminazole ring, is readily soluble in water and in absolute alcohol, forms characteristic benzoyl, phenyl isocyanate, and phenyl hydantoin derivatives, and yields certain characteristic color reactions, *e.g.*, with diazobenzene sulphonic acid, and with bromine water. J. S. H.

# THE FRANKLIN INSTITUTE.

*(Proceedings of the Stated Meeting held Wednesday, November 17, 1926.)*

The regular monthly meeting was called to order at eight-fifteen P.M. by Vice-President C. C. Tutwiler, who presided in the absence of the President.

The Secretary announced that the minutes of the last meeting had been published in full in the November number of the JOURNAL of the Institute and moved that the minutes be approved as printed. This motion was unanimously adopted.

The Secretary made a statement concerning the lectures on the Bartol Research Foundation, to be delivered on November seventeenth, nineteenth, and twenty-second, and issued a general invitation to the public to attend these lectures.

There was no further business.

The Chairman then presented Prof. Richard C. Tolman, of the California Institute of Technology, who delivered his first lecture on "Statistical Mechanics and Its Application to Physical-Chemical Problems."

Professor Tolman read a masterly and especially lucid lecture on this important development of modern physics.

The lecture was followed by an animated discussion.

The meeting adjourned at nine-twenty-eight P.M.

HOWARD MCCLENAHAN,
*Secretary.*

## COMMITTEE ON SCIENCE AND THE ARTS.

*(Abstract of Proceedings of Stated Meeting held Wednesday, November 3, 1926.)*

HALL OF THE COMMITTEE,
PHILADELPHIA, November 3, 1926.

MR. CLARENCE A. HALL *in the Chair.*

The following report was presented for final action:

No. 2852: Dr. Dayton C. Miller's Researches on Sound.

This report considered the work of Doctor Miller in this field under the following headings:

*The Flute and Flute Playing:* In this, attention is called to Doctor Miller's translation, with annotations, of Boehm's work on "The Flute and Flute Playing"; to the series of articles on the subject published by Doctor Miller; to the extensive library that he has collected on the flute; and to the collection of flutes that he has made to represent its development.

*The Phonodeik:* An instrument devised for the purpose of investigating the exceedingly small variations in air pressure which give to a musical tone its characteristic quality. It is also used for projecting the vibrations produced by sound waves and recording them photographically.

*The Effect of Horns and Diaphragms in Recording and Reproducing Devices.*

*Musical Instruments.*

*A Study of Vowel Sounds and Researches in Special Fields.*

The recommendation of the Sub-committee of Investigation that the Elliott Cresson Medal be awarded to Doctor Miller was adopted.

The following report was presented for first reading:

No. 2859: Waterless Gas Holder.

GEORGE A. HOADLEY,
*Secretary to Committee.*

## BARTOL RESEARCH FOUNDATION.

## A NEW THEORY OF LIGHT.

TO THE EDITORS:

Since the discovery of the Quantum Theory by Planck more than a quarter of a century ago, several attempts have been made to return to Newton's corpuscular theory of light. An outstanding difficulty has been the fact that the corpuscles, while possessing the same velocity, possess different amounts of energy and therefore different masses and hence seem to be of a different nature or at least particles of the same nature but of variable mass, depending on the wave-length of the light. Recently Prof. Gilbert Lewis (*Nat. Acad. Sci. Proc.*, **12**, pp. 22–29, Jan., 1926) has advanced the admirable hypothesis that the corpuscles of light are particles possessing the energy $h\nu$, the momentum $h\nu/c$ and the mass $h\nu/c^2$ (to be in agreement with classical mechanics, the momentum should be $2h\nu/c$ and the mass $2h\nu/c^2$), where $h$ is Planck's constant, $\nu$ the frequency and $c$ the velocity of light, but he leaves untouched the very important question as to the nature of these particles.

According to the modern views of the constitution of matter, it is very probable that the corpuscle of light may be one of the following: (*a*) Molecule, (*b*) atom, (*c*) electron, (*d*) sub-electron. However, we should expect, in analogy with α-particles (which are doubly charged helium atoms moving with very great speed), a molecule or an atom—with or without its accompanying electrons—when moving with the velocity of light, to have powerful ionizing effects which are, however, not shown by ordinary light, and hence we can eliminate a molecule and an atom; also as the existence of the sub-electron has not yet been demonstrated, there remains only the electron. Let us make the assumption (however absurd it may seem) that *light is a stream of electrons moving with the velocity of approximately 300,000 kilometres per second in vacuo* and see if it is possible to remove the very serious objection of a deflection of the electrons in an electromagnetic field.

The results of Kaufman (*Ann. d. Phys.*, 1906), Bucherer (*Phys. Zeits.*, 1908) and Hupka (*Deutsch. Phys. Ges.*, 1909) show clearly that the ratio of the charge $e$ of an electron to its mass $m$ decreases as the velocity of the particle increases; the results of Kaufman being in agreement with a complicated formula due to Abraham (see Mme. P. Curie, "Traité de Radioactivité," vol. 1, p. 66, and vol. 2, p. 42), which is founded on the assumption of rigidity

of the particle and uniform distribution of electricity on the particle, assumed spherical, whereas the results of Bucherer and Hupka agree with a much simpler expression, which is derived on the assumption of a contraction of the particle, originally spherical, in the sense of the motion, due to Lorentz, namely,

$$e/m = (e/m)_0 \sqrt{1-\beta^2}, \tag{1}$$

where $(e/m)_0$ represents the ratio of the charge to the mass when the electron is at rest and $\beta$ the ratio of the velocity of the particle to that of light. Lorentz's formula differs from that of Abraham appreciably only for small values of $\beta$, but it is very doubtful if the experimental results at that time were sufficiently accurate to support quantitatively these formulæ.

Hitherto physicists have regarded the electrical charge $e$ as constant and assigned the variation of the ratio $e/m$ to the mass $m$ which becomes according to Lorentz's formula, when we put $m_0$ equal to the mass of the particle at rest,

$$m = m_0/\sqrt{1-\beta^2}. \tag{2}$$

For $\beta = 1$, $m$ becomes infinite and the particle has then infinite energy as well, but light has finite energy and hence we are led to the alternate hypothesis of constant mass and variable charge which was used, nearly a century ago, by Gauss and Weber, but, due to the brilliant work of Maxwell on the assumption of an invariable charge, the hypothesis of Gauss and Weber was abandoned and gradually forgotten long before the rise of the electron theory of matter but has been recently resurrected by V. Bush (*Jour. Math. and Phys.*, Mass. Inst. of Technology, **5**, No. 3, pp. 129–157, March, 1926). Assuming the formula (1) correct, we have, when $e_0$ = charge of the particle at rest:

$$e = e_0\sqrt{1-\beta^2}. \tag{3}$$

Making use of only the first two terms of the expansion $\sqrt{1-\beta^2}$, that is, by putting $e = e_0(1-\beta^2/2)$, Bush has shown that all the laws of electric circuits can be derived as well as Sommerfeld's formula for the fine structure of spectral lines and approximately $10^{-23}$ cms. for the diameter of the atomic nucleus which is in agreement with that calculated from experimental results.

If $\beta = 1$, according to (3), $e = 0$; that is, we have the remarkable result, on this hypothesis, that *the charge on an electron moving with the speed of light falls to zero* and therefore no deflection in an electromagnetic field shall take place. Since the formula (3) has not yet been derived theoretically except from the formula (1) which in reality was derived by Lorentz on the assumption of a constant charge and a variable mass, let us replace (3) by the more general expression:

$$e = e_0 f(\beta) \tag{4}$$

where $f(0) = 1$, and $f(1) = 0$.

One difficulty, however, still remains: The energy $E$ of the particle is many millions times the energy of ordinary light. Taking the mass of the electron as approximately $9 \times 10^{-28}$ gram,

$$E = \frac{1}{2}mc^2 = \frac{1}{2} \times 9 \times 10^{-28} \times (3 \times 10^{10})^2 = 4 \times 10^{-7} \text{ erg}$$

which corresponds, on the basis of $e_0 = 4.774 \times 10^{-10}$ E.S.U., to:

$$V = 300 \times 4 \times 10^{-7} / (4.774 \times 10^{-10}) = 252{,}000 \text{ volts.}$$

This difficulty can easily be eliminated in the following manner: It has been shown (see, for example, W. C. McC. Lewis, "A System of Physical Chemistry," Vol. 1, Second Edition, p. 48) that a body of mechanical mass $m_k$, and having an electrical charge $e$, acts, when in motion, as if it possesses the mass $(m_k + 2e^2/3r)$ when $e$ is expressed in electromagnetic units or $(m_k + 2e^2/3rc^2)$ when $e$ is expressed in electrostatic units, where $r$ is the radius of the body assumed spherical and $c$ is the velocity of light. Putting $e = e_0 f(\beta)$, the mass $m$ of a body in motion becomes:

$$m = m_k + \frac{2e^2}{3rc^2} = m_k + \frac{2e_0^2}{3rc^2} \{f(\beta)\}^2 \tag{5}$$

Now when $\beta = 1$, $f(\beta) = 0$, and the mass becomes $m_k$.

As many physicists now consider mass and energy to be mutually convertible, let us make the assumption that a body having the mass $m_k$ and carrying the energy $h\nu$, acts as if it has the mass:

$$m_k + 2h\nu/c^2. \tag{6}$$

Combining (5) and (6), we have that the mass $m$ of a moving body of mechanical mass $m_k$, having an electrical charge $e$ and carrying the energy $h\nu$, is given by:

$$m = m_k + \frac{2h\nu}{c^2} + \frac{2e^2}{3rc^2} = m_k + \frac{2h\nu}{c^2} + \frac{2e_0^2}{3rc^2} \{f(\beta)\}^2. \tag{7}$$

When $\beta = 1$, then $m = m_k + 2h\nu/c^2$ and the energy $E = \frac{1}{2}m_k c^2 + h\nu$. When $\frac{1}{2}m_k c^2$ is negligible compared to $h\nu$, as in the case of high frequencies, then $E = h\nu$ (Planck's equation), but for low frequencies the energy of a ray of frequency $\nu$ should be greater than that given by Planck's equation. As yet we have no means of estimating $m_k$, but it is possible that we may be able to find its value by a more accurate study of the deviation of electrons in an electromagnetic field when they are moving with different speeds or perhaps by an accurate study of the energy of electromagnetic rays of very low frequencies.

If, however, $m_k$ is zero, that is, if all mechanical mass is electrical in origin, then the mass $m$ of a body having an electrical charge $e$ and carrying the energy $h\nu$, is given by:

$$m = \frac{2}{c^2}\left[h\nu + \frac{e^2}{3r}\right] = \frac{2}{c^2} \cdot \left[h\nu + \frac{e_0^2}{3r} \{f(\beta)\}^2\right]. \tag{8}$$

When $\beta = 1$, $m = 2h\nu/c^2$, which is the mass of Professor Lewis's particle of light (although Lewis mistakenly gave the value $h\nu/c^2$).

In the above discussion it has been assumed that the particle moves in a straight line, but if the particle is spinning or moving in a helix, some portion of the electron at any instant will be moving with a greater velocity than that of light, and in that case it will be necessary to consider supplementary masses, but I shall leave such cases for a later paper. In this connection it may be mentioned that according to Professor Eddington (*Nature*, 117, p. 652, May 8,

1926) the theory of relativity raises no objection to the case of the spinning electron where the periphery is supposed to move faster than light.

As pointed out by G. Lewis (*l. c.*), a corpuscular theory of light has difficulty in explaining interference. Lewis has offered an explanation, but it has been adversely criticized by Tolman and Smith (*Nat. Acad. Sci. Proc.*, **12**, pp. 343 and 508, 1926; see also Lewis, *Nat. Acad. Sci. Proc.*, **12**, p. 439, 1926).

The hypothesis of constant mass and variable electrical charge is, as pointed out by V. Bush (*l. c.*), contrary to the theory of relativity, but if Miller's experimental results of an ether drift be correct, the theory of relativity will have to be abandoned or at least seriously modified. Like every other corpuscular theory of light, this new theory of light does not require the existence of a hypothetical ether which very probably does not exist. I believe it is possible, on this new theory, to explain Michelson and Morley's results, but very difficult to explain Miller's results, which I shall leave for a later paper.

HERBERT J. BRENNEN, D.Sc.,
*Fellow, Bartol Research Foundation.*

November 15, 1926.

## MEMBERSHIP NOTES.

### ELECTIONS TO MEMBERSHIP.

*(Stated Meeting, Board of Managers, November 10, 1926.)*

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**CHANGES OF ADDRESS.**

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**Mr. F. H. Wyeth,** Address Unknown.

## LIBRARY NOTES.

**RECENT ADDITIONS.**

American Electrochemical Society.—Transactions, 1925. Volume 48. 1926.

BROCKMAN, C. J.—Electro-organic Chemistry. 1926.

CARTON, R., et DUMARTIN, P.—La Transformation de l'Énergie Électrique. I. Transformateurs. 1926.

Chemical Engineering Catalog. Eleventh annual edition. 1926.

CRANDALL, IRVING B.—Theory of Vibrating Systems and Sound. 1926.

DONKIN, C. T. B.—The Elements of Motor Vehicle Design. 1926.

FREUNDLICH, HERBERT.—Colloid and Capillary Chemistry. Translated from the third German edition by H. Stafford Hatfield. No date.

HAAS, ARTHUR.—Die Welt der Atome. 1926.

MARCOTTE, EDMOND.—Les Moteurs à Combustion. 1926.

Master Boiler Makers' Association.—Official Proceedings of the Seventeenth Annual Convention. 1926.

MICHAELIS, LEONOR.—Hydrogen Ion Concentration: Its Significance in the Biological Sciences and Methods for Its Determinations. Volume one: Principles of the Theory. Authorized translation from the second revised and enlarged German edition by W. A. Perlzweig. 1926.

Mineral Industry: Its Statistics, Technology and Trade during 1925. Volume 34. 1926.

MORSE, WITHROW.—Applied Biochemistry. 1925.

RIDEAL, ERIC KEIGHTLEY.—An Introduction to Surface Chemistry. 1926.

SAUVEUR, ALBERT.—The Metallography and Heat Treatment of Iron and Steel. Third edition. 1926.

SMITH, J. D. MAIN.—Chemistry and Atomic Structure. 1924.

Thomas' Register of American Manufacturers. Seventeenth edition, 1926–1927. 1926.

United States Department of Commerce.—Commerce Yearbook, 1925. 1926.

Van Nostrand's Chemical Annual. Sixth issue. 1926.

Wasserkraft Jahrbuch, 1925–1926. 1926.

## AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

Section M (The Engineering Section) of the Association will hold its meetings in the Hall of the Institute on Wednesday, December twenty-ninth. The day's meetings will be devoted to a symposium of the contributions that have been made by pure science to the advancement of engineering and industry. Dr. C. R. Richards, President of Lehigh University and President of the Section, will preside. The programme follows:

9:30 A.M.

Astronomy: Dr. Frank Schlesinger,
Director, The Yale University Observatory.

Biology: Dr. Vernon Kellogg,
Permanent Secretary, National Research Council.

Chemistry: Dr. Charles H. Herty,
President, Synthetic Organic Chemical Manufacturers' Association.

Economics: Dr. Joseph H. Willets,
Head of the Department of Industry, Wharton School of Finance and Commerce, University of Pennsylvania.

Geology: Dr. Heinrich Ries,
Professor, General and Economic Geology, Cornell University.

2:30 P.M.

Mathematics: Dr. G. A. Bliss,
Professor of Mathematics, University of Chicago.

Medical Science: Dr. Randle C. Rosenberger,
Professor of Preventative Medicine and Bacteriology, Jefferson Medical College.

Physics: Dr. R. A. Millikan,
Director, Norman Bridge Laboratory of Physics, California Institute of Technology.

Psychology: Dr. J. McKeen Cattell,
President of the Psychological Corporation, Editor of *Science*, the *Scientific Monthly*, and *School and Society*.

General Discussion.

6:30 P.M.

A subscription dinner of Section M will be held at the Bellevue-Stratford Hotel. Tickets for the dinner may be procured from the Secretary of the Engineers' Club.

After the dinner, the following addresses will be delivered:

"The Stimulation of Research in Pure Science That Has Resulted from the Needs of Engineers and of Industry," by Dr. W. R. Whitney, Director of the Research Laboratory of the General Electric Company, Schenectady, N. Y.

"Imhoff Tanks," by Mr. Harrison P. Eddy, Consulting Engineer, Boston, Mass., representing the American Society of Civil Engineers.

"The Scientific Aspects of Lighting," by Dr. M. Luckiesh, Director of the Lighting Research Laboratory, National Lamp Works, Cleveland, O.

"The Relationship between Science and the Study and Testing of Engineering Materials," by Mr. W. H. Fulweiler, United Gas Improvement Company, Philadelphia, Penna., representing A.S.T.M.

The members of The Franklin Institute are cordially invited to attend both the formal meetings and the dinner.

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**Phase-effect and the Localization of Sound.** H. BANISTER. (*Phil. Mag.*, Aug., 1926.)—To each ear of an auditor separately was brought an impure tone of about 220 double vibrations per second. The sounds were varied in intensity and in phase relation. From a study of results obtained with four persons, it is concluded that a change of 10 per cent. in total intensity is discernible. Furthermore, so long as the amplitudes of the two tones arriving at the two ears were kept unchanged, no change in the loudness of the resulting sensation was produced by shifting the phase difference between them. G. F. S.

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**DECEMBER 27–31, 1926**

IN THE

HALL OF THE INSTITUTE

ON

## RADIATIONS AND WAVE MOTIONS

SEEING WITHOUT VISIBLE LIGHT—X-RAYS—COLD LIGHT—DARK HEAT—RADIOS

BY

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Head of the Department of Physics
Johns Hopkins University

America's Most Inventive Experimenter

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The lectures are designed for young people who will be interested to learn something of what science is, and what science can do. They are similar to those which have been given for well nigh a century in the Royal Institution of London, where they have been received with enthusiasm.

Tickets for the course of four lectures may be procured from The Controller, The Franklin Institute. Price for the course, five dollars.

## BOOK REVIEWS.

ELECTRO-ORGANIC CHEMISTRY. By C. J. BROCKMAN, University of Georgia. xi–381 pages, 8vo. New York, John Wiley and Sons, Inc., 1926. Price, $5.

Electro-chemistry dates from more than a hundred years ago. Davy's well-known experiments upset the generally accepted views of the nature of the caustic alkalies and gave us the first two metals of low specific gravity. There is a tradition that when Davy showed the small specimen of potassium to Wollaston, the latter, poising it in hand, said, "How ponderous it is," misled by its evident metallic nature. The story may be untrue, but we can repeat it with the Italian apology, "Se non é vero é ben trovato"—"If not true, it is a lucky find." Early electrolytic experiments were made somewhat crudely, necessarily, from the limited data available and from the unsatisfactory apparatus for producing currents. Static phenomena had been observed in much detail for centuries. The younger generation, who know only the facility with which currents of all strengths and kinds can be obtained, cannot realize the difficulties when galvanic cells were the only source and when measuring instruments and even the electric units were not available.

In connection with the part that electrolysis plays to-day in chemistry and allied science, it is worth while to turn back to the earliest applications of it. According to Silliman's "Physics" (edition of 1866) it was in 1800 that Volta communicated to Sir Joseph Banks his discovery that enabled the production of a current, and shortly afterwards Nicholson and Carlisle constructed a "pile" of half-crowns (silver alloy), zinc and pasteboard wet with salt water. Observing gas-bubbles arising when the poles of this were immersed in water, Nicholson covered them with a glass tube filled with water and collected some of the gas. On the 2d of May, 1800, this experiment was made and the power of the electric current to decompose substances was discovered. The announcement of the results was, of course, followed by a repetition of the experiments with modifications in many laboratories. It is known to those familiar with the early history of this phase of the subject that the "contact theory" was held by many, to the effect that the current is due to the simple contact of dissimilar metals. The theory that the current is due to chemical action had been proposed some time before by Fabbroni, a countryman of Volta, and was taken up ardently by Davy. Later, as is also well known, Faraday determined the quantitative relations of the current and placed electrochemistry on a definite basis. The introduction of the theory of ionization still further elucidated the nature of electrolytic phenomena, and still later we have the principle of hydrion concentration, which has most important applications in biology and medicine, and is the subject of an extensive special literature.

The study of the electrolysis of organic compounds is not entirely new. Many years ago some experiments were made with familiar organic salts, but these were limited in scope and attracted but little attention. It needed the invention of convenient methods of producing currents of definite strength and methods of accurate measurement to raise the investigation to a satisfactory basis. Such requirements have only recently been attainable, and hence the data given in the book in hand cover a comparatively new field. The author

has searched the literature and has produced a work that presents a comprehensive and clear summary of the nature and effects of the application of electrolytic methods to many types of organic compounds.

HENRY LEFFMANN.

DIE WELT DER ATOME. By Dr. Arthur Haas, University of Vienna. xii–130 pages, numerous illustrations, 8vo, paper bound. Berlin and Leipzig, Walter de Gruyter and Company, 1926. Price, 4.80 marks.

This is a report of ten lectures on the modern theories of the atom and the arrangement in compounds. They were delivered before students of the several departments of the University in the summer semester of 1926. The lecturer adapted the language to the comprehension of those who are not trained in the higher technicalities of the science. The subject, however, is inherently complex and some familiarity with chemical and physical data is necessary for complete understanding.

The theory that matter though appearing continuous is really discrete, dates from very early times, having been formulated by Greek philosophers more than two thousand years ago. They must have reached their opinions by deduction, for there seems to be no reason to suppose that they had experimental methods. They coined from their own language the adjective "atomos" (indivisible) and expressed some views concerning atomic association which are strikingly like those entertained to-day. Unfortunately, the original literature is in great part lost and we are without means of knowing the details of their views. A later philosopher, Aristotle, acquired dominance in Greek thought and throughout a large part of the mediæval period was also the guide for the philosophy of Europe. The atomic theory lay unapplied for many centuries. Dalton in the early part of the nineteenth century revived it and it soon became a guiding principle in chemistry and physics, especially when aided by Berzelius' suggestions of the simple chemical symbols.

During the nineteenth century the general opinion was that the atoms are indivisible and inalterable and that each element is a distinct form of matter. Both these views have gone by the board. The atom has been broken up into smaller particles and the interconvertibility of the elements demonstrated in some cases and rendered probable in many others.

Doctor Haas's book is a very excellent summary of the present knowledge of atomic structure and of the arrangements of the atoms in space. In a natural spirit of triumph the question so often put by philosophers to scientists as to whether an atom had been seen is now answered in the affirmative. Modern research has demonstrated the actual existence of the groups of electrons and protons that constitute the unit of chemical action.

The topics are treated without mathematical illustration and the numerous and excellent illustrations enable the reader to obtain a clear and comprehensive notion of the present theories concerning the chemistry and physics of the minute structure of matter and also the methods by which the data have been obtained. Dalton and Faraday are mentioned as pioneers in the study of the subject, and the workers who subsequently carried on investigations are given proper credit. The book represents a very timely and satisfactory treatise on the newer phase of chemistry.

HENRY LEFFMANN.

LA TRANSFORMATION DE L'ÉNERGIE ÉLECTRIQUE. I. Transformateurs, par René Carton, Ingénieur E.M.I., et Pierre Dumartin, Ingénieur A. et M.I.E.G. Collection Armand Colin No. 78. iv–217 pages, 17 × 11 cm., paper. Paris, Librairie, Armand Colin, 1926. Price, paper, 9 francs 60, bound, 11 francs 90.

Simple in principle as is that very essential component of the alternating-current system of transmission, the transformer offers many and varied problems of design. The literature of the subject is abundant, and the object of this presentation of the subject is primarily to supply a brief but complete account of the principles of modern transformer design. Material progress in recent years has been made both in design and construction of transformers, especially with reference to the difficulties of prejudicial phenomena encountered under the modern conditions of large capacities, high tensions and transients, which are of little moment in apparatus of moderate capacities and tensions. The authors have borne these questions in mind in tracing out in proper sequence the principles, considerations of design and construction and the characteristics of modern static transformers. Rotary converters are reserved for a second volume.

The work should prove a valuable reference guide for the designer.

LUCIEN E. PICOLET.

HYDROGEN-ION CONCENTRATION. By Leoner Michaelis, M.D., University of Berlin. Volume I, Principles of the Theory. Authorized translation by William A. Perlzweig, M.A., Ph.D., Johns Hopkins University and Hospital. xiv–299 pages, 8vo, illustrated. Baltimore, The Williams and Wilkins Company, 1926. Price, $5.

When the first edition of this work appeared prior to the great war, the subject now so prominent in physics and chemistry and so largely applied in biology was attracting no very wide attention. The book contributed to the widening of the interest therein. The literature on hydrogen-ion is now extensive and growing at a high rate. Doctor Michaelis states that the data are now so voluminous that no one person can master the whole field. The scope had to be so enlarged that the second edition has been prepared in several parts. The first volume, now presented in English by Doctor Perlzweig, covers the physico-chemical principles, leaving to subsequent volumes the methodology and the applications to biologic problems.

Analytical chemists, who deal almost entirely with titration of acids and alkalies, have little interest in the question of "hydrion concentration" (as it is now judiciously proposed to term the factor). Those who deal with fluids in contact with living tissues are obliged constantly to consider the proportion of acid or alkali that is in the ionized state. It has recently been pointed out that, as a class, acids and alkalies are not highly ionized in ordinary solutions. Our views on the question are affected by the fact that ever since chemical substances were carefully studied, several familiar acids and alkalies happen to be unusually ionizable. Sulphuric, nitric and hydrochloric acids on one hand,

sodium and potassium hydroxide on the other, are exceptions to the general rule, giving under ordinary dilutions a high proportionate pH.

The second edition and its English translation will be a totally new book. The present volume places the principles upon a wider basis than in the first edition, due largely to the broadening of the field of physical chemistry, but especially to the great increase in the multiplicity of the applications of the data. Among these may be mentioned the work of several American scientists, notably Doctor Wherry, who has studied with care and in much detail the relation of the pH of the soil solution to the growth of plants. Valuable data concerning plant cultivation, and even protection from soil infections, have been obtained. All such data will be found in the volume, yet to appear, devoted to the applications of the phenomena.

The present volume is an excellent presentation of the whole question of hydrion concentration. Among the newer material are: Recent contributions to the activity theory of ionization; modification of the theory concerning the decomposition of ampholytes and the theory of oxidation-reduction potentials.

The translation has been very carefully done and the book puts into the hands of the English-speaking chemist a useful and comprehensive summary of the principles of a very important field of physical chemistry.

HENRY LEFFMANN.

LES MOTEURS À COMBUSTION. Par Edmond Marcotte, Ingénieur Conseil (I.C.F.), Professeur à l'École speciale des Travaux Publiques, Ancien Chef du Service des Ateliers de Moteurs de l'Aviation Militaire. Collection Armand Colin No. 77. iv–220 pages, 17 × 11 cm., illustrated, paper. Paris, Librairie, Armand Colin, 1926. Price, paper, 9 francs 60, bound, 11 francs 90.

When the Diesel engine made its first appearance about thirty years ago, it was expected that its fuel economy and simplicity of method of operation would quickly win for it a leading position in the field of power generation. It was soon found, however, that many practical difficulties incident to the high pressures and temperatures essential to its operation were to be overcome; a new technique was entailed in the design of these engines. These difficulties were duly met in the course of time and to-day the Diesel engine leads among power generators of least fuel consumption.

Mr. Marcotte's book, like his previous contribution to this series on a kindred subject, "Moteurs à Explosion," announced in these pages several months ago, contains a copious fund of technical information on the subject. He describes at adequate length the principles of operation and technical details of the Diesel engine and its modification, the semi-Diesel engine, which has been widely adapted among European fishing fleets. Historical notes on the development of these engines and statistical data on engines of European manufacture are features of unusual interest.

The work will appeal alike to those readers who seek technical data on the subject and those who require general information in condensed form.

LUCIEN E. PICOLET.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report 247, Pressure of Air on Coming to Rest from Various Speeds. By A. F. Zahm. 7 pages, illustrations, quarto. Washington, Government Printing Office, 1926. Price, five cents.

The text gives theoretical formulas from which is computed a table for the pressure of air on coming to rest from various speeds, such as those of aircraft and propeller blades. Pressure graphs are given for speeds from 1 cm. sec. up to those of swift projectiles.

The present treatment, slightly modified, was prepared for the Bureau of Aeronautics, Navy Department, February 17, 1926, and by it was submitted for publication to the National Advisory Committee for Aeronautics.

Report 237, Tests on Thirteen Navy Type Model Propellers. By W. F. Durand. 17 pages, illustrations, quarto. Washington, Government Printing Office, 1926. Price, ten cents.

The tests on these model propellers were conducted at Stanford University under research authorization of the Committee and were undertaken for the purpose of determining the performance coefficients and characteristics for certain selected series of propellers of form and type as commonly used in recent Navy designs.

The first series includes seven propellers of pitch ratio varying by 0.10 from 0.50 to 1.10, the area, form of blade, thickness, etc., representing an arbitrary standard propeller which had shown good results.

The second series covers changes in thickness of blade section, other things equal, and the third series, changes in blade area, other things equal.

These models are all of the standard 36-inch diameter employed in this laboratory.

## PUBLICATIONS RECEIVED.

*Electro-organic Chemistry*, by C. J. Brockman, Associate Professor of Chemistry, University of Georgia. 381 pages, illustrations, 8vo. New York, John Wiley and Sons, 1926. Price, five dollars.

*Theory of Vibrating Systems and Sound*, by Irving B. Crandall, Ph.D. 272 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1926. Price, five dollars.

*Thermodynamics for Students of Chemistry*, by C. N. Hinshelwood, M.A. 185 pages, illustrations, 12mo. New York, E. P. Dutton and Company, 1926. Price, $1.80.

*Introduction to Contemporary Physics*, by Karl K. Darrow, Ph.D. 453 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1926. Price, six dollars.

*Physics for Colleges*, by H. Horton Sheldon, Ph.D., C. V. Kent, Ph.D., Carl W. Miller, Ph.D., and Robert F. Paton, Ph.D. 655 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1926. Price, $3.75.

*Aide-Mémoire-Formulaire de la T. S. F. théorique et pratique*, par E. Pacoret, Ingénieur-Électricien. Edition 1926. 577 pages, illustrations, 12mo. Paris, Librairie Scientifique, Albert Blanchard. Price, thirty-two francs.

*Les Transformations de l'Énergie Électrique. I, Transformateurs*, par René

Carton et Pierre Dumartin. Preface de G. Darriens. 217 pages, illustrations, 16mo. Paris, Armand Colin, 1926. Price, nine francs 60.

*Hydrogen-ion Concentration. Its Significance in the Biological Sciences and Methods for Its Determinations,* by Leonor Michaelis, M.D., Professor in the University of Berlin. Volume one, Principles of the Theory. Authorized translation from the second revised and enlarged German edition, by William A. Perlzweig, M.A., Ph.D. 299 pages, illustrations, 8vo. Baltimore, The Williams and Wilkins Company, 1926. Price, five dollars.

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**Meteors and the Constitution of the Upper Air.** F. A. LINDEMANN. (*Nature*, Aug. 7, 1926.)—While meteors lead a short luminous life, though a dazzling one, and disappear before they reach the surface of the earth, meteorites, on the other hand, come all the way to the earth and are subject to chemical analysis. In nearly all cases they are found to be "mixtures of metallic iron-nickel and of an olivine-like glass consisting of a double silicate of magnesium and aluminum, the proportions ranging from pure metal to pure glass. It seems legitimate to infer that meteors are composed of the same materials." Meteors make their appearance in that portion of the atmosphere lying between 150 and 80 km. above the earth, and disappear at any height. Their brightness varies from that of brilliant fire-balls to that of such specimens as can be seen only by the help of the telescope; their duration is from a part of a second to 15 secs., and their speeds are from 10 to 100 km./sec. No meteor has been observed with certainty to lose speed.

Something may be learned about a meteor. Let such a body appear at a height of 100 km. and, after traversing a non-radial path of 60 km. in 1.5 secs., disappear at a height of 60 km. Further let it be so bright that to an observer 150 km. away it appear as a star of the first magnitude, *i.e.*, let it radiate during its luminous career $3.3 \times 10^{10}$ ergs. The author holds that, when a mass moving 40

km./sec. collides with air molecules, all its initial kinetic energy will be converted into the energy of radiation. He states that $3.3 \times 10^{10}$ ergs is possessed by a mass of only 6.25 milligrams with a speed of 40 km./sec. If the mass be iron, in spherical form, its diameter is 1.15 mm. (The reviewer calculates the mass to be only 4.1 mg., using the same energy and velocity.)

When the meteor reaches the high regions of the atmosphere, collisions with molecules of air are so rare that their effect is slight. As denser regions are attained, impacts are more frequent, and a cap of gas forms in front of the meteor. From this heat flows into the meteor and it begins to volatilize. It does not become visible until it has produced a body of vapor much larger than itself, since a particle of a millimetre in diameter could not be seen 100 km. away. It is possible to compute the density of the air where the cap formed. This has been done for 65 meteors whose speed, brightness, etc., were observed. Consideration of what takes place after the development of the gas cap renders it possible to calculate from a second point of view the density of the air at the height where luminosity begins. This, too, has been done for a large number of meteors and the results agree quite well with the products of the former calculation. Both methods agree in assigning much greater density to the atmosphere at high altitude than would follow on the assumption that the temperature of 220° Abs., observed at the elevation of 11 km. above the earth, persists to the upper reaches of the atmosphere. For instance, both methods unite in giving for the density of the air at 150 km. above the earth a value of roughly from $10^{-8}$ to $10^{-9}$ gm./c.c., whereas on the assumption just stated it comes out between $10^{-11}$ and $10^{-12}$ gm./c.c., only about 1/1000 as much. This discrepancy can be removed by postulating that above the height of 50 km. the temperature of the air rises to that of the surface of the earth or somewhat higher. Professor Lindemann adduces both observed facts and theoretical deductions in support of this supposititious increase of temperature. Four empirical facts are marshalled in support. Iron and olivine need to be at least at 2000° Abs. to form the gas cap necessary for luminosity. Meteors have been observed moving as slowly as 12 km./sec. To get to a final temperature of 2000° by reason of the adiabatic compression due to the motion of the meteor the air compressed must have had an initial temperature of 300°, which is 80° higher than that at the 11 km. level. In the second place scarcely any meteors disappear between the 50 and 60 km. levels, while many disappear both above and below these limits. Should a meteor in its course pass from a warmer to a cooler region, there is reason based on physical theory to believe that it would be unlikely to disappear there. Such a condition of temperatures would hold if the air grows warmer above the 11 km. level. In the third place, sound sources such as fog-horns and artil-

lery in action are surrounded by alternate zones of audibility and silence. This arrangement is explicable provided the velocity of sound for any reason increase at about 50 km. above the earth. Such an increase would take place were the air at this level warmer than at lower elevations. The fourth argument rests upon the existence of an electrically conducting layer that accounts for the anomalous propagation of electric waves. This layer seems to be 50 km. high at sunset and 80 km. high at sunrise. A reason can be given for the occurrence of this layer just where the increase of temperature is inferred to take place in the air.

From the side of theory the author considers the equilibrium temperature of the air resulting from the absorption of solar radiation by oxygen, water vapor, carbon dioxide and ozone. He finds that at a height of 60 km. the influence of ozone outweighs that of the other gases and would produce a temperature of about 300°.

Professor Lindemann wishes that his conclusions could be tested by explorations of the upper air, but fears that the difficulties are insuperable and the cost prohibitive. However, F. J. W. Whipple (*Nature*, Aug. 28, 1926) seems more hopeful of the success of such an endeavor. Perhaps we shall eventually explore this inaccessible region that is only 40 miles distant from any one of us.

G. F. S.


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